

thm_2EpatternMatches_2EPMATCH__EXTEND__BOTH__ID (TMdMcn1GSwtEzcfYfuJ5N6BKpsBQu56BHRh)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ecombin_2E_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define $c_2Ecombin_2E_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 9 We define $c_2Ecombin_2E_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2ES A_27a (A_27a^{A_27a})) A_27a$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (2)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (5)$$

Definition 10 We define $c_2EpatternMatches_2EPMATCH_INCOMPLETE$ to be $\lambda A_27a : \iota.(c_2Ebool_2EARB\ A_27a)$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (6)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b \in ((A_27a^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))})^{A_27b}) \quad (7)$$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\\ & (\forall V0v \in A_{.27b}. ((\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH } A_{.27a} \\ & A_{.27b}) V0v) (c_{.2E} \text{list}_{.2E} \text{NIL } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27a})^{A_{.27b}}))) = \\ & (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH}_{.2E} \text{INCOMPLETE } A_{.27a})) \wedge (\forall V1v \in \\ & A_{.27b}. (\forall V2r \in ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27a})^{A_{.27b}}). (\forall V3rs \in \\ & (\text{ty}_{.2E} \text{elist}_{.2E} \text{elist } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27a})^{A_{.27b}})). ((\\ & \text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH } A_{.27a} A_{.27b}) V1v) (\text{ap } (\text{ap } (\\ & c_{.2E} \text{elist}_{.2E} \text{ECONS } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27a})^{A_{.27b}})) V2r) V3rs)) = \\ & (\text{ap } (\text{ap } (\text{ap } (c_{.2E} \text{option}_{.2E} \text{option}_{.2E} \text{CASE } A_{.27a} A_{.27a}) (\text{ap } V2r V1v)) \\ & (\text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH } A_{.27a} A_{.27b}) V1v) V3rs)) \\ & (c_{.2E} \text{combin}_{.2E} \text{EI } A_{.27a})))))) \quad (15) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\\ & \forall V0v \in A_{.27a}. (\forall V1 \text{rows}_{.old} \in (\text{ty}_{.2E} \text{elist}_{.2E} \text{elist } (\\ & (\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27b})^{A_{.27a}})). (\forall V2 \text{rows}_{.new} \in \\ & (\text{ty}_{.2E} \text{elist}_{.2E} \text{elist } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27b})^{A_{.27a}})). (\forall V3r \in \\ & ((\text{ty}_{.2E} \text{option}_{.2E} \text{option } A_{.27b})^{A_{.27a}}). (((\text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH} \\ & A_{.27b} A_{.27a}) V0v) V1 \text{rows}_{.old}) = (\text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH} \\ & A_{.27b} A_{.27a}) V0v) V2 \text{rows}_{.new})) \Rightarrow ((\text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH} \\ & A_{.27b} A_{.27a}) V0v) (\text{ap } (\text{ap } (c_{.2E} \text{elist}_{.2E} \text{ECONS } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option} \\ & A_{.27b})^{A_{.27a}})) V3r) V1 \text{rows}_{.old})) = (\text{ap } (\text{ap } (c_{.2E} \text{patternMatches}_{.2E} \text{P} \text{MATCH} \\ & A_{.27b} A_{.27a}) V0v) (\text{ap } (\text{ap } (c_{.2E} \text{elist}_{.2E} \text{ECONS } ((\text{ty}_{.2E} \text{option}_{.2E} \text{option} \\ & A_{.27b})^{A_{.27a}})) V3r) V2 \text{rows}_{.new})))))) \end{aligned}$$