

thm_2EpatternMatches_2EPMATCH__EXTEND__OLD
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 8 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Eoption_2Eoption_ABS\ A.27a \in ((ty_2Eoption_2Eoption\ A.27a)^{(ty_2Esum_2Esum\ A.27a\ ty_2Eone_2Eone)}) \quad (5)$$

Definition 9 We define $c_2Eoption_2ESOME$ to be $\lambda A.27a : \iota. \lambda V0x \in A.27a. (ap\ (c_2Eoption_2Eoption_ABS\ A.27a)\ V0x)$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 12 We define c_2Esum_2EINR to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V0e \in A.27b. (ap\ (c_2Esum_2EABS\ A.27a\ A.27b)\ V0e)$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A.27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A.27a)\ (\lambda V0x \in A.27a. V0x))$

Definition 14 We define $c_2Ecombin_2EK$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0x \in A.27a. (\lambda V1y \in A.27b. V0x))$

Definition 15 We define $c_2Ecombin_2ES$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda A.27c : \iota. (\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Definition 16 We define $c_2Ecombin_2EI$ to be $\lambda A.27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A.27a\ (A.27a^{A.27a}))\ A.27a))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Eoption_2Eoption_CASE\ A.27a\ A.27b \in (((A.27b^{(A.27b^{A.27a})})^{A.27b})^{(ty_2Eoption_2Eoption\ A.27a)}) \quad (6)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (8)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ebool_2EARB\ A.27a \in A.27a \quad (9)$$

Definition 17 We define $c_2EpatternMatches_2EPMATCH_INCOMPLETE$ to be $\lambda A.27a : \iota. (c_2Ebool_2EARB\ A.27a)$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b \in ((A_27a^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})})})^{A_27b}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\ & (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a \\
& A_27b)\ V0v)\ (c_2Elist_2ENIL\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))) = \\
& (c_2EpatternMatches_2EPMATCH_INCOMPLETE\ A_27a))) \wedge (\forall V1v \in \\
& A_27b. (\forall V2r \in ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}). (\forall V3rs \in \\
& (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})). ((\\
& ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})\ V2r)\ V3rs)) = \\
& (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27a)\ (ap\ V2r\ V1v)) \\
& (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ V3rs)) \\
& (c_2Ecombin_2EI\ A_27a)))))) \\
& \hspace{15em} (20)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0v_old \in A_27a. (\forall V1v_new \in A_27b. \\
& (\forall V2rows_old \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27a})). (\forall V3rows_new \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b})). (\forall V4r_old \in ((ty_2Eoption_2Eoption\ A_27c)^{A_27a}). \\
& (((ap\ V4r_old\ V0v_old) = (c_2Eoption_2ENONE\ A_27c)) \Rightarrow (((ap\ (\\
& ap\ (c_2EpatternMatches_2EPMATCH\ A_27c\ A_27a)\ V0v_old)\ V2rows_old) = \\
& (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27c\ A_27b)\ V1v_new)\ V3rows_new)) \Rightarrow \\
& ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27c\ A_27a)\ V0v_old) \\
& (ap\ (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption\ A_27c)^{A_27a}) \\
& V4r_old)\ V2rows_old)) = (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& A_27c\ A_27b)\ V1v_new)\ V3rows_new))))))
\end{aligned}$$