thm_2EpatternMatches_2EPMATCH__FLATTEN__THM (TMTobXPMRahrxZD-CyeQA4PKtDyfpnrmTLch)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{3}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{5}$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{6}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{7}$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 12 *We define* c_2EpatternMatches_2EPMATCH__ROW__COND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0pat \in (A\_27b^{A\_27a}).\lambda V1guard \in (2^{A\_27a}).\lambda V2inp \in A\_27b.\lambda V3v \in A\_27a.(ap\ (ap$

Definition 13 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eoption_2Esome *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECO}$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{8}$$

Definition 18 *We define* c_2EpatternMatches_2EPMATCH__FLATTEN__FUN *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (A\_27c^{A\_27b}).\lambda V1g \in (2^{A\_27b}).\lambda V2row \in (((ty\_2Eoption\_2Eopti$

Let $c\_2Elist\_2ENIL : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{9}$$

Let $c\_2EpatternMatches\_2EPMATCH : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{A\_27b}) \tag{10}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{11}$$

Definition 19 *We define* c_2EpatternMatches_2EPMATCH__ROW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V$

Let $c\_2Elist\_2ECONS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{12}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{13}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Definition 20 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 21 *We define* c_2EpatternMatches_2EPMATCH__EQUIV__ROWS *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in A\_27a.\lambda V1rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27}$

Let $c\_2Elist\_2EMAP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{15}$$

Let $c\_2Elist\_2EEXISTS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{16}$$

Definition 22 *We define* c_2EpatternMatches_2EPMATCH__IS__EXHAUSTIVE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in A\_27a.\lambda V1rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a0 \in A\_27a.(\forall V1a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2a0\_27 \in A\_27a.(\forall V3a1\_27 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0a0)\ V1a1) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2a0\_27)\ V3a1\_27)) \Leftrightarrow ((V0a0 = V2a0\_27) \wedge (V1a1 = V3a1\_27)))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2))\ V2l3))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0v \in A\_27a.(\forall V1rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_ROWS}\ A\_27a\ A\_27b)\ V0v)\ V1rows)\ V1rows)))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0v \in A\_27a.(\forall V1rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V2rows2 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_ROWS}\ A\_27a\ A\_27b)\ V0v)\ V1rows1)\ V2rows2)) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ V0v)\ V1rows1) = (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ V0v)\ V2rows2)))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0v \in A\_27a.(\forall V1rows1a \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V2rows1b \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V3rows2a \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V4rows2b \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_ROWS}\ A\_27a\ A\_27b)\ V0v)\ V1rows1a)\ V2rows1b)) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_}\ A\_27a\ A\_27b)\ V0v)\ V3rows2a)\ V4rows2b)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_}\ A\_27a\ A\_27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V1rows1a)\ V3rows2a))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V2rows1b)\ V4rows2b)))))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0v \in A\_27a.(\forall V1p \in (A\_27a^{A\_27b}).\ (\forall V2g \in (2^{A\_27b}).(\forall V3rows \in (ty\_2Elist\_2Elist\ (\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})).((\forall V4x \in A\_27b.(p\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_IS\_\_EXHAUSTIVE}\ A\_27b\ A\_27c)\ V4x)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\lambda V5r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ V5r\ V4x)))\ V3rows)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_EQUIV\_\_ROWS}\ A\_27a\ A\_27c)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27b\ A\_27a)\ V1p)\ V2g)\ (\lambda V6x \in A\_27b.(ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V6x)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\lambda V7r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ V7r\ V6x)))\ V3rows)))))\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (\lambda V8r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_FLATTEN\_\_FUN}\ A\_27c\ A\_27b\ A\_27a)\ V1p)\ V2g)\ V8r)))\ V3rows))))))))) \tag{30}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0v \in A\_27a.(\forall V1p \in (A\_27a^{A\_27b}).(\forall V2g \in (2^{A\_27b}).(\forall V3rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a})).(\forall V4rows2 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a})).(\forall V5rows \in (ty\_2Elist\_2Elist\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})).((\forall V6x \in A\_27b.(p\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_IS\_\_EXHAUSTIVE}\ A\_27b\ A\_27c)\ V6x)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\lambda V7r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ V7r\ V6x)))\ V5rows)))) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ V3rows1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27b\ A\_27a)\ V1p)\ V2g)\ (\lambda V8x \in A\_27b.(ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V8x)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\lambda V9r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ V9r\ V8x)))\ V5rows)))))\ V4rows2))) = (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ V3rows1)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b})\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (\lambda V10r \in (((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})^{A\_27b}).(ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_FLATTEN\_\_FUN}\ A\_27c\ A\_27b\ A\_27a)\ V1p)\ V2g)\ V10r)))\ V5rows)))\ V4rows2))))))))))$$