

thm_2EpatternMatches_2EPMATCH_INTRO_CATCHALL (TMH68hk1zZbEYKJZUbKqdQKUah6jE88orAd)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2EpatternMatches_2EPMATCH_ROW_COND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0pat \in (A_27b^{A_27a}).\lambda V1guard \in (2^{A_27a}).\lambda V2inp \in A_27b.\lambda V3v \in A_27a.(ap (ap$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 9 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 11 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 12 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone))$

Definition 13 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 15 We define c_Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E40$

Definition 16 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 17 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap (ap (ap (c_Ebool_2ECOND$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Definition 18 We define $c_EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (7)$$

Let $c_2Elist_2ESNOC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ESNOC A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (8)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2EpatternMatches_2EPMATCH A_27a A_27b \in ((A_27a^{(ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_27a)^{A_27b}))})^{A_27b}) \quad (9)$$

Let $c_Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Ebool_2EARB A_27a \in A_27a \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0p \in (A_27b^{A_27a}).(\forall V1g \in (2^{A_27a}). \\ & (\forall V2r \in (A_27c^{A_27a}).(\forall V3v \in A_27b.(\forall V4rows \in \\ & (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27c)^{A_27b})).((\\ & \forall V5x \in A_27a.((ap\ V2r\ V5x) = (c_2Ebool_2EARB\ A_27c))) \Rightarrow ((\\ & ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27c\ A_27b)\ V3v)\ (ap\ (ap\ (\\ & c_2Elist_2ESNOC\ ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ (ap\ (ap\ \\ & (ap\ (c_2EpatternMatches_2EPMATCH_ROW\ A_27c\ A_27a\ A_27b)\ V0p)\ \\ & V1g)\ V2r))\ V4rows)) = (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27c \\ & A_27b)\ V3v)\ V4rows))))))))) \end{aligned} \quad (14)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0v \in A_27b.(\forall V1rows \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ & A_27a)^{A_27b})).((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b) \\ & V0v)\ V1rows) = (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b) \\ & V0v)\ (ap\ (ap\ (c_2Elist_2ESNOC\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})) \\ & (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_ROW\ A_27a\ A_27b\ A_27b) \\ & (\lambda V2_0 \in A_27b.V2_0))\ (\lambda V3_0 \in A_27b.c_2Ebool_2ET)) \\ & (\lambda V4_0 \in A_27b.(c_2Ebool_2EARB\ A_27a))))\ V1rows)))))) \end{aligned}$$