

thm_2EpatternMatches_2EPMATCH_IS_EXHAUSTIVE_LIFT (TMc3aNYggZ6PF9gqMezstaEVGtB3xJfzctf)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_2F})))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Elist_2EMAP} \\ & A. 27a \ A. 27b \in (((\text{ty_2Elist_2Elist } A. 27b)^{(\text{ty_2Elist_2Elist } A. 27a)})^{(A. 27b^{A-27a})}) \end{aligned} \quad (2)$$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (3)$$

Let `c_2Eoption_2EIS_SOME` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Eoption_2EIS_SOME } A. 27a \in (2^{(\text{ty_2Eoption_2Eoption } A. 27a)}) \quad (4)$$

Let `c_2Elist_2EEXISTS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Elist_2EEXISTS } A. 27a \in (2^{(\text{ty_2Elist_2Elist } A. 27a)})^{(2^{A-27a})} \quad (5)$$

Definition 8 We define `c_2EpatternMatches_2EPMATCH_IS_EXHAUSTIVE` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0v \in A_{.27a}. \lambda V1rs \in (ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_{.27b})^{A_{.27a}}))$.

Let `c_2Eoption_2EOPTION_MAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow c_2Eoption_2EOPTION_MAP A_{.27a} A_{.27b} \in (((ty_2Eoption_2Eoption A_{.27b})^{(ty_2Eoption_2Eoption A_{.27a})})^{(A_{.27b}^{A_{.27a}})}) \quad (6)$$

Definition 9 We define `c_2EpatternMatches_2EPMATCH_ROW_LIFT` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda A_{.27c} :$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{.27a}. (p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\forall A_{.27a}. nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow (\\ & \quad \forall V0P \in (2^{A_{.27b}}). (\forall V1f \in (A_{.27b}^{A_{.27a}}). (\forall V2l \in \\ & \quad (ty_2Elist_2Elist A_{.27a}). ((p (ap (ap (c_2Elist_2EEXISTS A_{.27b}) \\ & V0P) (ap (ap (c_2Elist_2EMAP A_{.27a} A_{.27b}) V1f) V2l))) \Leftrightarrow (p (ap (ap \\ & (c_2Elist_2EEXISTS A_{.27a}) (\lambda V3x \in A_{.27a}. (ap V0P (ap V1f V3x))) \\ & V2l)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in (ty_2Eoption_2Eoption \\
& A_27a). ((p (ap (c_2Eoption_2EIS_SOME\ A_27b) (ap (ap (c_2Eoption_2EOPTION_MAP \\
& A_27a\ A_27b)\ V0f)\ V1x))) \Leftrightarrow (p (ap (c_2Eoption_2EIS_SOME\ A_27a) \\
& \quad V1x))))))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1v \in A_27c. \\
& \quad (\forall V2rows \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27c})). \\
& \quad ((p (ap (ap (c_2EpatternMatches_2EPMATCH_IS_EXHAUSTIVE\ A_27c \\
& A_27a)\ V1v)\ V2rows))) \Rightarrow (p (ap (ap (c_2EpatternMatches_2EPMATCH_IS_EXHAUSTIVE \\
& A_27c\ A_27b)\ V1v) (ap (ap (c_2Elist_2EMAP\ ((ty_2Eoption_2Eoption \\
& A_27a)^{A_27c}) ((ty_2Eoption_2Eoption\ A_27b)^{A_27c})) (ap (c_2EpatternMatches_2EPMATCH_ROW_LIFT \\
& \quad A_27c\ A_27b\ A_27a)\ V0f))\ V2rows))))))
\end{aligned}$$