# thm_2EpatternMatches_2EPMATCH__REMOVE__ARB__NO__OVE (TMWa2E6msqhYxL9FkhCAic8bjgwa3qKiGtJ)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (2)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (3)$$

**Definition 7** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 8** *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

**Definition 9** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{5}$$

**Definition 10** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{6}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{7}$$

**Definition 11** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{8}$$

**Definition 12** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{9}$$

**Definition 13** *We define* c_2EpatternMatches_2EPMATCH__ROW__COND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0pat \in (A\_27b^{A\_27a}).\lambda V1guard \in (2^{A\_27a}).\lambda V2inp \in A\_27b.\lambda V3v \in A\_27a.(ap\ (ap$

**Definition 14** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

**Definition 15** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

**Definition 16** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 17** *We define* c_2Eoption_2Esome *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (ap$ (c_2Ebool_2EC

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{10}$$

**Definition 18** *We define* c_2EpatternMatches_2EPMATCH__ROW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V$

Let $c\_2Elist\_2ESNOC : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{11}$$

Let $c\_2EpatternMatches\_2EPMATCH : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{A\_27b}) \tag{12}$$

Let $c\_2Elist\_2ECONS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{13}$$

Let $c\_2Elist\_2EAPPEND : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}True)) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( p\ V0t))))))) \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \land (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{22}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \land ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ V1l) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0p \in (A\_27b^{A\_27a}).\ (\forall V1g \in (2^{A\_27a}).(\forall V2r \in (A\_27c^{A\_27a}).(\forall V3p\_27 \in (A\_27b^{A\_27d}).(\forall V4g\_27 \in (2^{A\_27d}).(\forall V5r\_27 \in (A\_27c^{A\_27d}).\ (\forall V6rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})).(\forall V7rows2 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})).(\forall V8rows3 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})).(\forall V9v \in A\_27b.(((\forall V10x \in A\_27a.((\ (V9v = (ap\ V0p\ V10x)) \wedge (p\ (ap\ V1g\ V10x))) \Rightarrow (\exists V11x\_27 \in A\_27d.\ (((ap\ V0p\ V10x) = (ap\ V3p\_27\ V11x\_27)) \wedge (p\ (ap\ V4g\_27\ V11x\_27)))))) \wedge \ ((\forall V12x \in A\_27a.(\forall V13x\_27 \in A\_27d.(((V9v = (ap\ V0p\ V12x)) \wedge (((ap\ V0p\ V12x) = (ap\ V3p\_27\ V13x\_27)) \wedge ((p\ (ap\ V1g\ V12x)) \wedge (p\ (ap\ V4g\_27\ V13x\_27))))) \Rightarrow ((ap\ V2r\ V12x) = (ap\ V5r\_27\ V13x\_27))))) \wedge \ (\forall V14x \in A\_27a.(((V9v = (ap\ V0p\ V14x)) \wedge (p\ (ap\ V1g\ V14x))) \Rightarrow \ (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\lambda V15row \in ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27c))\ (ap\ V15row\ (ap\ V0p\ V14x)))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c))))\ V7rows2)))))) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V9v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ V6rows1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27a\ A\_27b)\ V0p)\ V1g)\ V2r))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((\ ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ V7rows2)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27d\ A\_27b)\ V3p\_27)\ V4g\_27)\ V5r\_27))\ V8rows3))))) = (ap\ (\ ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V9v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ V6rows1)\ V7rows2))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (\ ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27d\ A\_27b)\ V3p\_27)\ V4g\_27)\ V5r\_27))\ V8rows3))))))))))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0v \in A\_27b.(\forall V1rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V0v)\ V1rows) = (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27a\ A\_27b\ A\_27b)\ (\lambda V2\_\_0 \in A\_27b.V2\_\_0))\ (\lambda V3\_\_0 \in A\_27b.\mathsf{c\_2Ebool\_2ET}))\ (\lambda V4\_\_0 \in A\_27b.(c\_2Ebool\_2EARB\ A\_27a))))\ V1rows))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1h \in A\_27a.(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3l3 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1h)\ V2l2)))\ V3l3) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l2)\ V3l3)))))))) \tag{28}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0v \in A\_27a.(\forall V1p \in (A\_27a^{A\_27b}).(\forall V2g \in (2^{A\_27b}).(\forall V3r \in (A\_27c^{A\_27b}).(\forall V4rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a})).(\forall V5rows2 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a})).(((\forall V6x \in A\_27b.((ap\ V3r\ V6x) = (c\_2Ebool\_2EARB\ A\_27c))) \wedge (\forall V7x \in A\_27b.(((V0v = (ap\ V1p\ V7x)) \wedge (p\ (ap\ V2g\ V7x))) \Rightarrow (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (\lambda V8row \in ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27c))\ (ap\ V8row\ (ap\ V1p\ V7x)))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c))))\ V5rows2))))) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ V4rows1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27b\ A\_27a)\ V1p)\ V2g)\ V3r))\ V5rows2))) = (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27a}))\ V4rows1)\ V5rows2))))))))))$$