

thm_2EpatternMatches_2EPMATCH_ROWS_DROP_SUBSUME
(TMS-
DNzuEvrKVTm7n8wdZJ6qysEdLLzF92TB)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (2)$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (3)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 8 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (7)$$

Definition 9 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E\ V0t))$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (the\ (\lambda x. x \in A \wedge \neg P\ x)))$

Definition 15 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 16 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 17 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a)\ (the\ (\lambda x. x \in A \wedge \neg P\ x)))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (8)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (9)$$

Definition 18 We define `c_2EpatternMatches_2EPMATCH_INCOMPLETE` to be $\lambda A_{.27a} : \iota.(c_2Ebool_2EARB\ A_{.27a})$.

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow c_2Elist_2ENIL\ A_{.27a} \in (ty_2Elist_2Elist\ A_{.27a}) \quad (10)$$

Let `c_2Eoption_2EIS_SOME` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow c_2Eoption_2EIS_SOME\ A_{.27a} \in (2^{(ty_2Eoption_2Eoption\ A_{.27a})}) \quad (11)$$

Let `c_2Elist_2ELIST_TO_SET` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow c_2Elist_2ELIST_TO_SET\ A_{.27a} \in ((2^{A_{.27a}})^{(ty_2Elist_2Elist\ A_{.27a})}) \quad (12)$$

Definition 19 We define `c_2Ebool_2EIN` to be $\lambda A_{.27a} : \iota.(\lambda V0x \in A_{.27a}.(\lambda V1f \in (2^{A_{.27a}}).(ap\ V1f\ V0x)))$

Definition 20 We define `c_2Ebool_2E_3F` to be $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_{.27a}\ V0P))))$

Definition 21 We define `c_2Ebool_2ECOND` to be $\lambda A_{.27a} : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_{.27a}.(\lambda V2t2 \in A_{.27a}.(V0t \Rightarrow V1t1 \Rightarrow V2t2))))$

Let `c_2EpatternMatches_2EPMATCH` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow c_2EpatternMatches_2EPMATCH\ A_{.27a}\ A_{.27b} \in ((A_{.27a}^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_{.27a})^{A_{.27b}}))})^{A_{.27b}}) \quad (13)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow c_2Elist_2ECONS\ A_{.27a} \in (((ty_2Elist_2Elist\ A_{.27a})^{(ty_2Elist_2Elist\ A_{.27a})})^{A_{.27a}}) \quad (14)$$

Let `c_2Elist_2EAPPEND` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow c_2Elist_2EAPPEND\ A_{.27a} \in (((ty_2Elist_2Elist\ A_{.27a})^{(ty_2Elist_2Elist\ A_{.27a})})^{(ty_2Elist_2Elist\ A_{.27a})}) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \vee \\ (p\ V2C)) \wedge (p\ V0A)) \Leftrightarrow (((p\ V1B) \wedge (p\ V0A)) \vee ((p\ V2C) \wedge (p\ V0A)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ (p\ V1B)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in 2. (\forall V1P_27 \in 2. (\forall V2Q \in 2. (\forall V3Q_27 \in \\ 2. (((p\ V2Q) \Rightarrow ((p\ V0P) \Leftrightarrow (p\ V1P_27))) \wedge ((p\ V1P_27) \Rightarrow ((p\ V2Q) \Leftrightarrow (p\ V3Q_27)))))) \Rightarrow \\ (((p\ V0P) \wedge (p\ V2Q)) \Leftrightarrow ((p\ V1P_27) \wedge (p\ V3Q_27)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p (ap\ V0f\ V2x)))) \Leftrightarrow (p (ap\ V0f\ V1v)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI A_27a)\ V0x) = V0x)) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((p (ap (ap (c_2Elist_2EVERY\ A_27a)\ V0P)\ V1l)) \Leftrightarrow (\forall V2e \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V2e)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l))) \Rightarrow (p (ap\ V0P\ V2e)))))))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0e \in A_27a. (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist\ A_27a). ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0e)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap (ap (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2)))) \Leftrightarrow ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0e)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l1))) \vee (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0e)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ V2l2)))))))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A_27a. (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist\ A_27a). ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap (ap (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap (c_2Elist_2ELIST_TO_SET\ A_27a)\ V3t)))))))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. (V0opt = (ap (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ & A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\ & (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ & A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE \\ & A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ & A_27a)\ (c_2Eoption_2ENONE\ A_27a))) \Leftrightarrow False)) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eoption_2Eoption \\ & A_27a). (\neg(p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27a)\ V0x))) \Leftrightarrow (V0x = \\ & (c_2Eoption_2ENONE\ A_27a)))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b. ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a \\ & A_27b)\ V0v)\ (c_2Elist_2ENIL\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))) = \\ & (c_2EpatternMatches_2EPMATCH_INCOMPLETE\ A_27a))) \wedge (\forall V1v \in \\ & A_27b. (\forall V2r \in ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}). (\forall V3rs \in \\ & (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})). ((\\ & ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})\ V2r)\ V3rs)) = \\ & (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27a)\ (ap\ V2r\ V1v)) \\ & (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ V3rs)) \\ & (c_2Ecombin_2EI\ A_27a)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0v \in A.27a. (\forall V1rows1 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& \quad \quad A.27b)^{A.27a})). (\forall V2rows2 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& \quad \quad A.27b)^{A.27a})). ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A.27b\ A.27a) \\
& \quad \quad V0v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption\ A.27b)^{A.27a})) \\
& \quad \quad V1rows1)\ V2rows2)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A.27b)\ (ap\ (c_2Ebool_2E_3F \\
& \quad \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}))\ (\lambda V3r \in ((ty_2Eoption_2Eoption \\
& \quad \quad A.27b)^{A.27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}))\ V3r)\ (ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}))\ V1rows1)))\ (ap\ (c_2Eoption_2EIS_SOME \\
& \quad \quad A.27b)\ (ap\ V3r\ V0v))))))\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& \quad \quad A.27b\ A.27a)\ V0v)\ V1rows1))\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& \quad \quad A.27b\ A.27a)\ V0v)\ V2rows2))))))
\end{aligned}$$

(49)

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\
& \quad A.27a). (\forall V1h \in A.27a. (\forall V2l2 \in (ty_2Elist_2Elist \\
& \quad \quad A.27a). (\forall V3l3 \in (ty_2Elist_2Elist\ A.27a). ((ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad \quad A.27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad \quad A.27a)\ V1h)\ V2l2)))\ V3l3) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A.27a)\ V0l1) \\
& \quad \quad (ap\ (ap\ (c_2Elist_2ECONS\ A.27a)\ V1h)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad \quad A.27a)\ V2l2)\ V3l3))))))
\end{aligned}$$

(50)

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0r1 \in ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V1r2 \in \\
& \quad ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V2rows1 \in (ty_2Elist_2Elist \\
& \quad (ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V3rows2 \in (ty_2Elist_2Elist \\
& \quad (ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V4rows3 \in (ty_2Elist_2Elist \\
& \quad ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (\forall V5v \in A_27a. (\\
& \quad ((\forall V6x \in A_27b. ((ap\ V0r1\ V5v) = (ap\ (c_2Eoption_2ESOME\ A_27b) \\
& \quad V6x)) \Rightarrow ((ap\ V1r2\ V5v) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V6x)))) \wedge (\\
& \quad (p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27b)\ (ap\ V0r1\ V5v))) \Rightarrow (p\ (ap\ (ap \\
& \quad (c_2Elist_2EVERY\ ((ty_2Eoption_2Eoption\ A_27b)^{A_27a})\ (\lambda V7row \in \\
& \quad ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D \\
& \quad (ty_2Eoption_2Eoption\ A_27b))\ (ap\ V7row\ V5v))\ (c_2Eoption_2ENONE \\
& \quad A_27b))))\ V3rows2)))) \Rightarrow ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& \quad A_27b\ A_27a)\ V5v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V2rows1)\ (ap\ (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V0r1)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V3rows2)\ (ap\ (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V1r2)\ V4rows3)))))) = (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& \quad A_27b\ A_27a)\ V5v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V2rows1)\ V3rows2))\ (ap\ (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A_27b)^{A_27a})\ V1r2)\ V4rows3)))))))))
\end{aligned}$$