

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (1)$$

Definition 16 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (2)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (3)$$

Definition 18 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (5)$$

Definition 19 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eone_2Eone))$

Definition 20 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 21 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 22 We define $c_2Eoption_2Esome$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ (c_2Ebool_2E_7E\ V0P)\ c_2Eone_2Eone)\ c_2Eone_2Eone)\ V0P)$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Definition 23 We define $c_2EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0x \in A_27a.(ap\ (ap\ (ap\ (c_2Ebool_2E_7E\ V0x)\ c_2Eone_2Eone)\ c_2Eone_2Eone)\ V0x)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b \in ((A_27a^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))})^{A_27b}) \quad (10)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (11)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (12)$$

Definition 24 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in \\ & (2^{A-27a}). ((\exists V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in \\ & A_27a.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ (ap\ V1Q\ V3x)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c.2Emin_2E_40 \\ & A_27a)\ V0P)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). ((\neg(\forall V1x \in \\ & A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). ((\neg(\exists V1x \in \\ & A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ & 2^{A-27a}). (((p\ V0P) \vee (\exists V2x \in A_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in \\ & A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ & (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ & (p\ V0A)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ & (p\ V0A)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ & (p\ V1B)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (38)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \Rightarrow (39)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{27a}.(\forall V3x_{27} \in A_{27a}.(\forall V4y \in A_{27a}.(\forall V5y_{27} \in A_{27a}.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_{27})) \wedge ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_{27})))))) \Rightarrow ((ap \ (ap \ (ap \ (c_{2Ebool_2ECOND} \ A_{27a} \ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_{2Ebool_2ECOND} \ A_{27a} \ V1Q) \ V3x_{27} \ V5y_{27})))))) \Rightarrow (40)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1a \in A_{27a}.((\exists V2x \in A_{27a}.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))) \Rightarrow (41)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0f \in (2^{A_{27a}}).(\forall V1v \in A_{27a}.((\forall V2x \in A_{27a}.((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ (ap \ V0f \ V1v)))) \Rightarrow (42)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}).(\forall V1x \in A_{27a}.(\exists V2y \in A_{27b}.(p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}).(\forall V4x \in A_{27a}.(p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x)))))) \Rightarrow (43)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap \ (c_{2Ecombin_2EI} \ A_{27a} \ V0x) = V0x)) \Rightarrow (44)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0opt \in (ty_{2Eoption_2Eoption} \ A_{27a}).((V0opt = (c_{2Eoption_2ENONE} \ A_{27a})) \vee (\exists V1x \in A_{27a}.(V0opt = (ap \ (c_{2Eoption_2ESOME} \ A_{27a} \ V1x)))))) \Rightarrow (45)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow ((\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1x \in A_{27a}.((ap \ (ap \ (c_{2Eoption_2EOPTION_MAP} \ A_{27a} \ A_{27b} \ V0f) \ (ap \ (c_{2Eoption_2ESOME} \ A_{27a} \ V1x)) = (ap \ (c_{2Eoption_2ESOME} \ A_{27b} \ (ap \ V0f \ V1x)))))) \wedge (\forall V2f \in (A_{27b}^{A_{27a}}).((ap \ (ap \ (c_{2Eoption_2EOPTION_MAP} \ A_{27a} \ A_{27b} \ V2f) \ (c_{2Eoption_2ENONE} \ A_{27a})) = (c_{2Eoption_2ENONE} \ A_{27b})))))) \Rightarrow (46)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a.((p\ (ap\ (c.2Eoption_2EIS_SOME\ A.27a)\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c.2Eoption_2EIS_SOME\ A.27a)\ (c.2Eoption_2ENONE\ A.27a))) \Leftrightarrow False)) \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1x \in A.27a. \\ (\forall V2y \in A.27a.(((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty.2Eoption_2Eoption\ A.27a))\ V0P)\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V1x))\ (c.2Eoption_2ENONE\ A.27a)) = (c.2Eoption_2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty.2Eoption_2Eoption\ A.27a))\ V0P)\ (c.2Eoption_2ENONE\ A.27a))\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V1x)) = (c.2Eoption_2ENONE\ A.27a)) \Leftrightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty.2Eoption_2Eoption\ A.27a))\ V0P)\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V1x))\ (c.2Eoption_2ENONE\ A.27a)) = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\ (((ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ (ty.2Eoption_2Eoption\ A.27a))\ V0P)\ (c.2Eoption_2ENONE\ A.27a))\ (ap\ (c.2Eoption_2ESOME\ A.27a)\ V1x)) = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = V2y)))))) \quad (48) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in (A.27b^{A.27a}).(\forall V1x \in (ty.2Eoption_2Eoption\ A.27a).(\forall V2y \in A.27b.(((ap\ (ap\ (c.2Eoption_2EOPTION_MAP\ A.27a\ A.27b)\ V0f)\ V1x) = (ap\ (c.2Eoption_2ESOME\ A.27b)\ V2y)) \Leftrightarrow (\exists V3z \in A.27a.((V1x = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V3z)) \wedge (V2y = (ap\ V0f\ V3z)))))) \quad (49) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0r1 \in ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (\forall V1r2 \in \\
& \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (\forall V2rows1 \in (ty_2Elist_2Elist \\
& \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (\forall V3rows2 \in (ty_2Elist_2Elist \\
& \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (\forall V4rows3 \in (ty_2Elist_2Elist \\
& \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (\forall V5v \in A.27a. (\\
& \quad ((\forall V6x \in A.27b. ((ap\ V0r1\ V5v) = (ap\ (c.2Eoption_2ESOME\ A.27b) \\
& \quad V6x)) \Rightarrow ((ap\ V1r2\ V5v) = (ap\ (c.2Eoption_2ESOME\ A.27b)\ V6x)))) \wedge (\\
& \quad (p\ (ap\ (c.2Eoption_2EIS_SOME\ A.27b)\ (ap\ V0r1\ V5v))) \Rightarrow (p\ (ap\ (ap \\
& \quad (c.2Elist_2EVERY\ ((ty_2Eoption_2Eoption\ A.27b)^{A.27a})\ (\lambda V7row \in \\
& \quad ((ty_2Eoption_2Eoption\ A.27b)^{A.27a}). (ap\ (ap\ (c.2Emin_2E_3D \\
& \quad (ty_2Eoption_2Eoption\ A.27b))\ (ap\ V7row\ V5v))\ (c.2Eoption_2ENONE \\
& \quad A.27b))))\ V3rows2)))) \Rightarrow ((ap\ (ap\ (c.2EpatternMatches_2EPMATCH \\
& \quad A.27b\ A.27a)\ V5v)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V2rows1)\ (ap\ (ap\ (c.2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V0r1)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V3rows2)\ (ap\ (ap\ (c.2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V1r2)\ V4rows3)))))) = (ap\ (ap\ (c.2EpatternMatches_2EPMATCH \\
& \quad A.27b\ A.27a)\ V5v)\ (ap\ (ap\ (c.2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ (ap\ (ap\ (c.2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V2rows1)\ V3rows2))\ (ap\ (ap\ (c.2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& \quad A.27b)^{A.27a})\ V1r2)\ V4rows3))))))))) \\
& \hspace{15em} (50)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \quad (55)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{65}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0p \in (A_27b^{A_27a}). \\
& (\forall V1g \in (2^{A_27a}). (\forall V2r \in (A_27c^{A_27a}). (\forall V3p_27 \in \\
& (A_27b^{A_27d}). (\forall V4g_27 \in (2^{A_27d}). (\forall V5r_27 \in (A_27c^{A_27d}). \\
& (\forall V6rows1 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b})). (\forall V7rows2 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b})). (\forall V8rows3 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b})). (\forall V9v \in A_27b. (((\forall V10x \in A_27a. ((\\
& (V9v = (ap\ V0p\ V10x)) \wedge (p\ (ap\ V1g\ V10x))) \Rightarrow (\exists V11x_27 \in A_27d. \\
& (((ap\ V0p\ V10x) = (ap\ V3p_27\ V11x_27)) \wedge (p\ (ap\ V4g_27\ V11x_27)))))) \wedge \\
& ((\forall V12x \in A_27a. (\forall V13x_27 \in A_27d. (((V9v = (ap\ V0p \\
& V12x)) \wedge (((ap\ V0p\ V12x) = (ap\ V3p_27\ V13x_27)) \wedge ((p\ (ap\ V1g\ V12x)) \wedge \\
& (p\ (ap\ V4g_27\ V13x_27)))))) \Rightarrow ((ap\ V2r\ V12x) = (ap\ V5r_27\ V13x_27)))))) \wedge \\
& (\forall V14x \in A_27a. (((V9v = (ap\ V0p\ V14x)) \wedge (p\ (ap\ V1g\ V14x))) \Rightarrow \\
& (p\ (ap\ (ap\ (c_2Elist_2EVERY\ ((ty_2Eoption_2Eoption\ A_27c)^{A_27b})) \\
& (\lambda V15row \in ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}). (ap\ (ap\ (c_2Emin_2E_3D \\
& (ty_2Eoption_2Eoption\ A_27c))\ (ap\ V15row\ (ap\ V0p\ V14x)))\ (c_2Eoption_2ENONE \\
& A_27c))))\ V7rows2)))))) \Rightarrow ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\
& A_27c\ A_27b)\ V9v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b}))\ V6rows1)\ (ap\ (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption \\
& A_27c)^{A_27b}))\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_ROW \\
& A_27c\ A_27a\ A_27b)\ V0p)\ V1g)\ V2r))\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((\\
& ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ V7rows2)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_ROW \\
& A_27c\ A_27d\ A_27b)\ V3p_27)\ V4g_27)\ V5r_27))\ V8rows3)))))) = (ap\ (\\
& ap\ (c_2EpatternMatches_2EPMATCH\ A_27c\ A_27b)\ V9v)\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ V6rows1)\ V7rows2))\ (ap \\
& (ap\ (c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption\ A_27c)^{A_27b}))\ (\\
& ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_ROW\ A_27c\ A_27d\ A_27b) \\
& V3p_27)\ V4g_27)\ V5r_27))\ V8rows3)))))))))
\end{aligned}$$