

thm\_2EpatternMatches\_2EPMATCH\_ROW\_COND\_NOT\_EX\_C  
(TMZhAu7TxBWCmNo7eUPRp55o6BtcaWX4VdM)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{3}$$

**Definition 9** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_Esum\_2EABS\_$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 10** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_$

**Definition 11** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x)$

**Definition 12** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 13** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE A\_27a A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption A\_27a)}) \quad (6)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (7)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (8)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ebool\_2EARB A\_27a \in A\_27a \quad (9)$$

**Definition 14** We define  $c\_2EpatternMatches\_2EPMATCH\_INCOMPLETE$  to be  $\lambda A\_27a : \iota. (c\_2Ebool\_2EARB A\_27a).$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (10)$$

Let  $c\_2EpatternMatches\_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2EpatternMatches\_2EPMATCH A\_27a A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist ((ty\_2Eoption\_2Eoption A\_27a)^{A\_27b}))})^{A\_27b}) \quad (11)$$

Let  $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (12)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (13)$$

**Definition 15** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone))$

**Definition 16** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 17** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\ A\_27a\ A\_27b)\ V0e)$

**Definition 18** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_2EABS\ A\_27a)\ (c\_2Eoption\_2ENONE\ A\_27a))$

**Definition 19** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ V2t)\ V1t2))\ V0t1))$

**Definition 20** We define  $c\_2EpatternMatches\_2EPMATCH\_ROW\_COND\_NOT\_EX\_OR\_EQ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in A\_27a.\lambda V1r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).\lambda V2rows \in (ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}.$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap (c\_2Ecombin\_2El A\_27a) V0x) = V0x)) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) V0P) (c\_2Elist\_2ENIL A\_27a))) \Leftrightarrow False)) \wedge (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist A\_27a).((p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) V1P) (ap (ap (c\_2Elist\_2ECONS A\_27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \vee (p (ap (ap (c\_2Elist\_2EEXISTS A\_27a) V1P) V3t)))))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption A\_27a).((V0opt = (c\_2Eoption\_2ENONE A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap (c\_2Eoption\_2ESOME A\_27a) V1x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b. (\forall V1f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b)\ (c\_2Eoption\_2ENONE\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A\_27a. (\forall V3v \in A\_27b. (\forall V4f \in (A\_27b^{A\_27a}). ((ap\ (ap \\ & (ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b)\ (ap\ (c\_2Eoption\_2ESOME \\ & A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. (((ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME \\ & A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\neg((c\_2Eoption\_2ENONE \\ & A\_27a) = (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Eoption\_2ETHE \\ & A\_27a)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x)) = V0x)) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b. ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a \\ & A\_27b)\ V0v)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))) = \\ & (c\_2EpatternMatches\_2EPMATCH\_INCOMPLETE\ A\_27a)) \wedge (\forall V1v \in \\ & A\_27b. (\forall V2r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}). (\forall V3rs \in \\ & (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})). (( \\ & ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ (ap\ (ap\ ( \\ & c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})\ V2r)\ V3rs)) = \\ & (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27a)\ (ap\ V2r\ V1v)) \\ & (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ V3rs)) \\ & (c\_2Ecombin\_2EI\ A\_27a)))))) \end{aligned} \quad (33)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0i \in A\_27a. (\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}). \\ & (\forall V2r\_27 \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}). (\forall V3rows \in \\ & (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})). (( \\ & \neg((ap\ V2r\_27\ V0i) = (c\_2Eoption\_2ENONE\ A\_27b))) \Rightarrow ((p\ (ap\ (ap\ (ap \\ & (c\_2EpatternMatches\_2EPMATCH\_ROW\_COND\_NOT\_EX\_OR\_EQ \\ & A\_27a\ A\_27b)\ V0i)\ V1r)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption \\ & A\_27b)^{A\_27a})\ V2r\_27)\ V3rows))) \Leftrightarrow ((\neg((ap\ V1r\ V0i) = (c\_2Eoption\_2ENONE \\ & A\_27b))) \Rightarrow ((ap\ V1r\ V0i) = (ap\ V2r\_27\ V0i)))))) \end{aligned}$$