

thm_2EpatternMatches_2EPMATCH_ROW_CONG (TMRv3WpDyYYdy5XaG8StyjYbMwtsRqoXyHV)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$) of type $\iota \Rightarrow \iota$.

Definition 4 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 16 We define $c_2EpatternMatches_2EPMATCH_ROW_COND$ to be
 $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0pat \in (A_27b^{A_27a}). \lambda V1guard \in (2^{A_27a}). \lambda V2inp \in A_27b. \lambda V3v \in A_27a. (ap (ap$

Definition 17 We define $c_2Eoption_2Esome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Definition 18 We define $c_2EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V1x))) \Rightarrow (p V0t)) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).(((\exists V2x \in A_27a.(p(ap V0P V2x))) \wedge (\forall V3x \in A_27a.((p(ap V0P V3x)) \Rightarrow (p(ap V1Q V3x)))))) \Rightarrow (p(ap V1Q (ap(c_2Emin_2E_40 A_27a) V0P)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p(ap V0P V2x)))))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in 2.(\forall V1P_{.27} \in 2.(\forall V2Q \in 2.(\forall V3Q_{.27} \in \\ & 2.(((p V2Q) \Rightarrow ((p V0P) \Leftrightarrow (p V1P_{.27}))) \wedge ((p V1P_{.27}) \Rightarrow ((p V2Q) \Leftrightarrow (p V3Q_{.27})))))) \Rightarrow \\ & (((p V0P) \wedge (p V2Q)) \Leftrightarrow ((p V1P_{.27}) \wedge (p V3Q_{.27})))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in \\ & A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (\\ & ap V0f V1v)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0opt \in (ty_{.2Eoption} _2Eoption \\ & A_{.27a}).((V0opt = (c_{.2Eoption} _2ENONE A_{.27a})) \vee (\exists V1x \in A_{.27a}. \\ & (V0opt = (ap (c_{.2Eoption} _2ESOME A_{.27a}) V1x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in \\ & A_{.27a}.(((ap (c_{.2Eoption} _2ESOME A_{.27a}) V0x) = (ap (c_{.2Eoption} _2ESOME \\ & A_{.27a}) V1y)) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1x \in A_{.27a}.((ap (ap (c_{.2Eoption} _2EOPTION_MAP \\ & A_{.27a} A_{.27b}) V0f) (ap (c_{.2Eoption} _2ESOME A_{.27a}) V1x)) = (ap (c_{.2Eoption} _2ESOME \\ & A_{.27b}) (ap V0f V1x)))))) \wedge (\forall V2f \in (A_{.27b}^{A_{.27a}}).((ap (ap (c_{.2Eoption} _2EOPTION_MAP \\ & A_{.27a} A_{.27b}) V2f) (c_{.2Eoption} _2ENONE A_{.27a}) = (c_{.2Eoption} _2ENONE \\ & A_{.27b})))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A_27a. \\
& (\forall V2y \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE \\
& A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (\\
& ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE \\
& A_27a))\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (c_2Eoption_2ENONE \\
& A_27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE \\
& A_27a)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a)) \\
& V0P)\ (c_2Eoption_2ENONE\ A_27a))\ (ap\ (c_2Eoption_2ESOME\ A_27a) \\
& V1x)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\
& p\ V2r))) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\
& \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (38)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & \quad nonempty \ A.27c \Rightarrow (\forall V0p \in (A.27b^{A.27a}). (\forall V1p.27 \in (\\ & \quad \quad A.27b^{A.27a}). (\forall V2g \in (2^{A.27a}). (\forall V3g.27 \in (2^{A.27a}). \\ & \quad \quad (\forall V4r \in (A.27c^{A.27a}). (\forall V5r.27 \in (A.27c^{A.27a}). (\forall V6v \in \\ & \quad \quad A.27b. (\forall V7v.27 \in A.27b. (((V0p = V1p.27) \wedge ((V6v = V7v.27) \wedge \\ & \quad \quad ((\forall V8x \in A.27a. ((V6v = (ap \ V0p \ V8x)) \Rightarrow ((p \ (ap \ V2g \ V8x)) \Leftrightarrow (p \ (\\ & \quad \quad \quad ap \ V3g.27 \ V8x)))))) \wedge (\forall V9x \in A.27a. (((V6v = (ap \ V0p \ V9x)) \wedge (\\ & \quad \quad \quad p \ (ap \ V2g \ V9x)) \Rightarrow ((ap \ V4r \ V9x) = (ap \ V5r.27 \ V9x)))))) \Rightarrow ((ap \ (ap \ (ap \\ & \quad \quad \quad (ap \ (c.2EpatternMatches.2EPMATCH_ROW \ A.27c \ A.27a \ A.27b) \ V0p) \\ & \quad \quad \quad V2g) \ V4r) \ V6v) = (ap \ (ap \ (ap \ (ap \ (c.2EpatternMatches.2EPMATCH_ROW \\ & \quad \quad \quad A.27c \ A.27a \ A.27b) \ V1p.27) \ V3g.27) \ V5r.27) \ V7v.27)))))))))) \end{aligned}$$