

thm_2EpatternMatches_2EPMATCH_ROW_EQ_AUX (TMEnDRhEs4g5q3CPmzj6BtEUsrBRD5o1tK6)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.\lambda V0x)$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A.\lambda c^{A-27b})^{A-27a})$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A.\lambda a : \iota.(ap (ap (c_2Ecombin_2ES A.\lambda a (A.\lambda a^{A-27a}) A.$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a.nonempty A \Rightarrow c_2Eoption_2EIS_SOME A \Rightarrow a \in (2^{(ty_2Eoption_2Eoption A)}) \quad (2)$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (3)$$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 10 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (6)$$

Definition 11 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 13 We define $c_2Ebool_2E_2$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E_40\ A_27a)\ V2t2)\ V1t1)\ V0t))$

Definition 15 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2))$

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 18 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 19 We define $c_2EpatternMatches_2EPMATCH_ROW_COND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0pat \in (A_27b^{A_27a}).\lambda V1guard \in (2^{A_27a}).\lambda V2inp \in A_27b.\lambda V3v \in A_27a.(ap\ (ap\ (c_2Emin_2E_40\ A_27a)\ V3v)\ V2inp)\ V1guard)\ V0pat)$

Definition 20 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ (c_2Ebool_2E_2)\ V0P)\ V0P)\ V0P)$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in ((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)\ (A_27b^{A_27a})}) \quad (7)$$

Definition 21 We define $c_2EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0pat \in (A_27b^{A_27a}).\lambda V1guard \in (2^{A_27a}).\lambda V2inp \in A_27b.\lambda V3v \in A_27a.(ap\ (ap\ (c_2Emin_2E_40\ A_27a)\ V3v)\ V2inp)\ V1guard)\ V0pat)$

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ V1t2)\ V0t1))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. ((p \ V0t) \vee \neg(p \ V0t))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((p \ V0t) \Rightarrow False) \Rightarrow \neg(p \ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p \ V0t) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))) \quad (18)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).(((\exists V2x \in A_27a.(p (ap V0P V2x))) \wedge (\forall V3x \in A_27a.((p (ap V0P V3x)) \Rightarrow (p (ap V1Q V3x)))))) \Rightarrow (p (ap V1Q (ap (c_2Emin_2E_40 A_27a) V0P)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1b \in 2.(\forall V2x \in A_27a. \\ (\forall V3y \in A_27a.((ap \ V0f \ (ap \ (ap \ (ap \ (c.2Ebool_2ECOND \ A_27a) \\ V1b) \ V2x) \ V3y)) = (ap \ (ap \ (ap \ (c.2Ebool_2ECOND \ A_27b) \ V1b) \ (ap \ V0f \\ V2x)) \ (ap \ V0f \ V3y))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c.2Ebool_2ECOND \ A_27a) \\ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c.2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\ V5y_27))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in \\ A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (\\ ap \ V0P \ V1a)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in \\ A_27a.((\forall V2x \in A_27a.((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ (\\ ap \ V0f \ V1v)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE\ A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b)\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ (ap\ V0f\ V1x)))) \wedge (\forall V2f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b)\ V2f)\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE\ A_27b)))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27a)\ (c_2Eoption_2ENONE\ A_27a))) \Leftrightarrow False)) \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A_27a. \\
& (\forall V2y \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE \\
& A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (\\
& ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE \\
& A_27a))\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (c_2Eoption_2ENONE \\
& A_27a)) \Leftrightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE \\
& A_27a)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a)) \\
& V0P)\ (c_2Eoption_2ENONE\ A_27a))\ (ap\ (c_2Eoption_2ESOME\ A_27a) \\
& V1x)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Eoption_2ESOME\ A_27a) \\
& (\lambda V0x \in A_27a.c_2Ebool_2EF)) = (c_2Eoption_2ENONE\ A_27a))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{47}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{53}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0p \in (A_27a^{A_27b}). \\
& (\forall V1g \in (2^{A_27b}). (\forall V2p_27 \in (A_27a^{A_27c}). (\forall V3g_27 \in \\
& (2^{A_27c}). (\forall V4r \in (A_27d^{A_27b}). (\forall V5r_27 \in (A_27d^{A_27c}). \\
& (((\forall V6i \in A_27a. ((\exists V7x \in A_27b. (p \ (ap \ (ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW_COND \\
& A_27b \ A_27a) \ V0p) \ V1g) \ V6i) \ V7x))) \Leftrightarrow (\exists V8x_27 \in A_27c. (p \ (ap \\
& (ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW_COND \ A_27c \ A_27a) \\
& V2p_27) \ V3g_27) \ V6i) \ V8x_27)))))) \wedge (\forall V9x \in A_27b. (\forall V10x_27 \in \\
& A_27c. (((ap \ V0p \ V9x) = (ap \ V2p_27 \ V10x_27)) \wedge ((p \ (ap \ V1g \ V9x)) \wedge (\\
& p \ (ap \ V3g_27 \ V10x_27)))) \Rightarrow ((ap \ V4r \ V9x) = (ap \ V5r_27 \ V10x_27)))))) \Rightarrow \\
& ((ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW \ A_27d \ A_27b \ A_27a) \\
& V0p) \ V1g) \ V4r) = (ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW \\
& A_27d \ A_27c \ A_27a) \ V2p_27) \ V3g_27) \ V5r_27))))))
\end{aligned}$$