

thm_2EpatternMatches_2EPMATCH_ROW_EQ_NONE (TMc4KCgbjfxGeUo2GPaATJq1ACDnFHpKShZ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone))$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 14 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E40$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap$

Definition 16 We define $c_2EpatternMatches_2EPMATCH_ROW_COND$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0pat \in (A_27b^{A_27a}). \lambda V1guard \in (2^{A_27a}). \lambda V2inp \in A_27b. \lambda V3v \in A_27a. (ap (ap$

Definition 17 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Definition 18 We define $c_2EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg (\exists V1x \in A_27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg (p (ap V0P V2x)))))) \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A.27a. \\
& (\forall V2y \in A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (\\
& ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a))\ V0P)\ (c.2Eoption.2ENONE \\
& A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (c.2Eoption.2ENONE \\
& A.27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a)) \\
& V0P)\ (c.2Eoption.2ENONE\ A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a) \\
& V1x)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27a^{A.27b}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27b). (((ap\ (ap\ (c.2Eoption.2EOPTION_MAP\ A.27b\ A.27a)\ V0f) \\
& V1x) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b))) \wedge \\
& (((c.2Eoption.2ENONE\ A.27a) = (ap\ (ap\ (c.2Eoption.2EOPTION_MAP \\
& A.27b\ A.27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE\ A.27b))))))
\end{aligned} \tag{12}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0p \in (A.27c^{A.27b}). (\forall V1g \in (2^{A.27b}). \\
& (\forall V2r \in (A.27a^{A.27b}). (\forall V3i \in A.27c. (((ap\ (ap\ (ap\ (\\
& ap\ (c.2EpatternMatches.2EPMATCH_ROW\ A.27a\ A.27b\ A.27c)\ V0p) \\
& V1g)\ V2r)\ V3i) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (\forall V4x \in A.27b. \\
& (\neg(p\ (ap\ (ap\ (ap\ (ap\ (c.2EpatternMatches.2EPMATCH_ROW_COND \\
& A.27b\ A.27c)\ V0p)\ V1g)\ V3i)\ V4x))))))))))
\end{aligned}$$