

# thm\_2EpatternMatches\_2EPMATCH\_ROW\_LIFT\_THM (TMWx3zPtcsFRatMTwJskohPoKgoM2tgxbUq)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A))$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 8** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone$

**Definition 9** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2EF$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (3)$$

**Definition 12** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_2Esum\_2EABS$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (ap$

**Definition 14** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_2Esum\_2EABS$

**Definition 15** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption$

**Definition 16** We define  $c\_2EpatternMatches\_2EPMATCH\_ROW\_COND$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0pat \in (A\_27b^{A\_27a}). \lambda V1guard \in (2^{A\_27a}). \lambda V2inp \in A\_27b. \lambda V3v \in A\_27a. (ap (ap$

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 18** We define  $c\_2Eoption\_2ESome$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{A\_27a}). (ap (ap (ap (c\_2Ebool\_2ECO$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP A\_27a A\_27b \in (((ty\_2Eoption\_2Eoption A\_27b)^{(ty\_2Eoption\_2Eoption A\_27a)})^{(A\_27b^{A\_27a})}) \quad (6)$$

**Definition 19** We define  $c\_2EpatternMatches\_2EPMATCH\_ROW$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V$

**Definition 20** We define  $c\_2EpatternMatches\_2EPMATCH\_ROW\_LIFT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1r \in ((ty\_2Eoption\_2Eoption A\_27c)^{A\_27a}). (\lambda V2$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption \\ A\_27a).((V0opt = (c\_2Eoption\_2ENONE\ A\_27a)) \vee (\exists V1x \in A\_27a. \\ (V0opt = (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.(((ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x) = (ap\ (c\_2Eoption\_2ESOME \\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V0f)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)) = (ap\ (c\_2Eoption\_2ESOME \\ A\_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V2f)\ (c\_2Eoption\_2ENONE\ A\_27a)) = (c\_2Eoption\_2ENONE \\ A\_27b)))) \end{aligned} \quad (12)$$

### Theorem 1

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). \\ (\forall V1p \in (A\_27d^{A\_27c}).(\forall V2g \in (2^{A\_27c}).(\forall V3r \in \\ (A\_27a^{A\_27c}).((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW\_LIFT \\ A\_27d\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW \\ A\_27a\ A\_27c\ A\_27d)\ V1p)\ V2g)\ V3r)) = (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW \\ A\_27b\ A\_27c\ A\_27d)\ V1p)\ V2g)\ (\lambda V4x \in A\_27c.(ap\ V0f\ (ap\ V3r\ V4x)))))))))) \end{aligned}$$