

thm_2EpatternMatches_2EREDUNDANT__ROWS__INFOS__CONJ
(TMb8bkf7Tm5crHi4qb9sEQkXnkVhQ27PWNV)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let $c_2Elist_2EZIP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EZIP A_27a A_27b \in ((ty_2Elist_2Elist (ty_2Epair_2Eprod A_27a A_27b))^{(ty_2Epair_2Eprod (ty_2Elist_2Elist A_27a A_27b))}) \quad (3)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EMAP A_27a A_27b \in (((ty_2Elist_2Elist A_27b)^{(ty_2Elist_2Elist A_27a)})^{(A_27b^{A_27a})}) \quad (4)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$.
Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (5)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2E_21) x y)$.
Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (6)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (7)$$

Definition 10 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$.
Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (8)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40) ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone)$.

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS_sum) e)$.

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (12)$$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Elist_2EVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EVERY A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(2^{A_27a})}) \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (14)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EEL A_27a \in ((A_27a^{(ty_2Elist_2Elist A_27a)})^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (16)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 16 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E40$

Definition 17 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist A_27a)}) \quad (20)$$

Definition 18 We define $c_2EpatternMatches_2EPMATCH_ROW_REDUNDANT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in A_27a.\lambda V1rs \in (ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_27b)^{A_27a})$

Definition 19 We define $c_2EpatternMatches_2EIS_REDUNDANT_ROWS_INFO$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0v \in A_27a.\lambda V1rows \in (ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_27b)^{A_27a})$

Let $c_2Elist_2EMAP2 : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow c_2Elist_2EMAP2 A_27a A_27b A_27c \in (((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist A_27c)})^{(ty_2Elist_2Elist A_27b)})^{((A_27a^{A_27c})^{A_27b})} \end{aligned} \quad (21)$$

Definition 20 We define $c_2EpatternMatches_2EREDUNDANT_ROWS_INFOS_CONJ$

to be $\lambda V0ip1 \in (ty_2Elist_2Elist\ 2).\lambda V1ip2 \in (ty_2Elist_2Elist\ 2).(ap\ (ap\ (ap\ (c_2Elist_2EMAP2\ 2\ 2\ 2)$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge ((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (30)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \quad (31)$$

$$(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\quad$$

$$\forall V0l \in (ty_2Elist_2Elist \ A_27a).(\forall V1f \in (A_27b^{A_27a}).$$

$$((ap \ (c_2Elist_2ELENGTH \ A_27b) \ (ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \ V1f) \ V0l)) = (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l)))) \quad (32)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\quad$$

$$\forall V0n \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist \ A_27a).((p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V0n) \ (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1l))) \Rightarrow (\forall V2f \in (A_27b^{A_27a}).((ap \ (ap \ (c_2Elist_2EEL \ A_27b) \ V0n) \ (ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \ V2f) \ V1l)) = (ap \ V2f \ (ap \ (ap \ (c_2Elist_2EEL \ A_27a) \ V0n) \ V1l)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\quad$$

$$\forall V0l1 \in (ty_2Elist_2Elist \ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \ A_27b).(((ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l1) = (ap \ (c_2Elist_2ELENGTH \ A_27b) \ V1l2)) \Rightarrow (((ap \ (c_2Elist_2ELENGTH \ (ty_2Epair_2Eprod \ A_27a \ A_27b)) \ (ap \ (c_2Elist_2EZIP \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ (ty_2Elist_2Elist \ A_27a) \ (ty_2Elist_2Elist \ A_27b)) \ V0l1) \ V1l2))) = (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l1)) \wedge ((ap \ (c_2Elist_2ELENGTH \ (ty_2Epair_2Eprod \ A_27a \ A_27b)) \ (ap \ (c_2Elist_2EZIP \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ (ty_2Elist_2Elist \ A_27a) \ (ty_2Elist_2Elist \ A_27b)) \ V0l1) \ V1l2))) = (ap \ (c_2Elist_2ELENGTH \ A_27b) \ V1l2)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\quad$$

$$\forall V0l1 \in (ty_2Elist_2Elist \ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist \ A_27b).(\forall V2n \in ty_2Enum_2Enum.(((ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l1) = (ap \ (c_2Elist_2ELENGTH \ A_27b) \ V1l2)) \wedge (p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V2n) \ (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l1))) \Rightarrow ((ap \ (ap \ (c_2Elist_2EEL \ (ty_2Epair_2Eprod \ A_27a \ A_27b)) \ V2n) \ (ap \ (c_2Elist_2EZIP \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Epair_2E_2C \ (ty_2Elist_2Elist \ A_27a) \ (ty_2Elist_2Elist \ A_27b)) \ V0l1) \ V1l2))) = (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ (ap \ (ap \ (c_2Elist_2EEL \ A_27a) \ V2n) \ V0l1)) \ (ap \ (ap \ (c_2Elist_2EEL \ A_27b) \ V2n) \ V1l2)))))) \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1l2 \in \\
& (ty_2Elist_2Elist\ A_27b). (((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l1) = \\
& (ap\ (c_2Elist_2ELENGTH\ A_27b)\ V1l2)) \Rightarrow (\forall V2f \in ((A_27c^{A_27b})^{A_27a}). \\
& ((ap\ (ap\ (ap\ (c_2Elist_2EMAP2\ A_27c\ A_27a\ A_27b)\ V2f)\ V0l1)\ V1l2) = \\
& (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27c) \\
& (ap\ (c_2Epair_2EUNCURRY\ A_27a\ A_27b\ A_27c)\ V2f)))\ (ap\ (c_2Elist_2EZIP \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ A_27a) \\
& (ty_2Elist_2Elist\ A_27b))\ V0l1)\ V1l2)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\
& A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\
& A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\
& (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{37}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0v \in A_27a. (\forall V1rows \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\
& A_27b)^{A_27a})). (\forall V2c \in 2. (\forall V3infos \in (ty_2Elist_2Elist \\
& 2). (\forall V4c_27 \in 2. (\forall V5infos_27 \in (ty_2Elist_2Elist \\
& 2). ((p\ (ap\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EIS_REDUNDANT_ROWS_INFO \\
& A_27a\ A_27b)\ V0v)\ V1rows)\ V2c)\ V3infos)) \Rightarrow (((ap\ (c_2Elist_2ELENGTH \\
& 2)\ V5infos_27) = (ap\ (c_2Elist_2ELENGTH\ 2)\ V3infos)) \Rightarrow (p\ (ap\ (\\
& ap\ (ap\ (ap\ (c_2EpatternMatches_2EIS_REDUNDANT_ROWS_INFO \\
& A_27a\ A_27b)\ V0v)\ V1rows)\ (ap\ (ap\ c_2Ebool_2E_5C_2F\ V2c)\ V4c_27)) \\
& (ap\ (ap\ c_2EpatternMatches_2EREDUNDANT_ROWS_INFOS_CONJ \\
& V3infos)\ V5infos_27))))))))))
\end{aligned}$$