

# thm\_2Epoly\_2EORDER (TMKw8KnqFkuAmaeJjk3cyyh63jaN8khZfAQ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)\ ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)\ ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal) \tag{5}$$

Let  $c\_2Erealx\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (6)$$

Let  $c\_2Erealx\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_ABS\_CLASS \in (ty\_2Erealx\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \quad (7)$$

**Definition 6** We define  $c\_2Erealx\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 7** We define  $c\_2Erealx\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (8)$$

Let  $c\_2Epoly\_2Epoly\_mul : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_mul \in (((ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)})^{(ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)})^{(ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)} \quad (9)$$

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a\ P)\ V0Q)))$

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ t1\ t2))\ (\lambda V2t \in 2.t1\ t2))$

**Definition 11** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ P)\ V0Q)))$

Let  $c\_2Epoly\_2Epoly : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly \in ((ty\_2Erealx\_2Ereal)^{ty\_2Erealx\_2Ereal})^{(ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)} \quad (10)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (14)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$   
Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ENIL\ A.27a \in (ty\_2Elist\_2Elist\ A.27a) \quad (15)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega ga \quad (16)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 14** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 15** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Erealx\_2Etreax\_neg : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (18)$$

**Definition 17** We define  $c\_2Erealx\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.(ap\ c\_2Erealx\_2Ereal$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Elist\_2ECONS\ A.27a \in (((ty\_2Elist\_2Elist\ A.27a)^{(ty\_2Elist\_2Elist\ A.27a)})^{A.27a}) \quad (19)$$

Let  $c\_2Epoly\_2Epoly\_exp : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_exp \in (((ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)^{ty\_2Enum\_2Enum})^{(ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal)}) \quad (20)$$

**Definition 18** We define  $c\_2Epoly\_2Epoly\_divides$  to be  $\lambda V0p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealx\_2Ereal).$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 19** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E.3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

**Definition 21** We define  $c\_2Epoly\_2Epoly\_order$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.\lambda V1p \in (ty\_2Elist\_2E$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))\ (22)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((\forall V2x \in \\
& A\_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x))) \Rightarrow (V0f = V1g))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in \\
& A\_27a.((\forall V2y \in A\_27a.((p \ (ap \ V0P \ V2y)) \Leftrightarrow (V2y = V1x))) \Rightarrow ((ap \\
& (c\_2Emin\_2E\_40 \ A\_27a) \ V0P) = V1x))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((p (ap (c.2Ebool.2E.3F.21 A.27a) (\lambda V1x \in A.27a.(ap V0P V1x)))) \Leftrightarrow ((\exists V2x \in A.27a.(p (ap V0P V2x))) \wedge (\forall V3x \in A.27a.(\forall V4y \in A.27a.((p (ap V0P V3x)) \wedge (p (ap V0P V4y))) \Rightarrow (V3x = V4y)))))) \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \quad \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ & A.27b.(p\ (ap\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).( \\ & \quad \forall V4x \in A.27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c.2Epoly\_2Epoly\ ( \\ & c.2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0x) = (ap\ c.2Ereal\_2Ereal\_of\_num \\ & c.2Enum\_2E0))) \wedge (\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in \\ & (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V3x \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c.2Epoly\_2Epoly\ (ap\ (ap\ (c.2Elist\_2ECONS\ ty\_2Erealax\_2Ereal \\ & V1h)\ V2t))\ V3x) = (ap\ (ap\ c.2Erealax\_2Ereal\_add\ V1h)\ (ap\ (ap\ c.2Erealax\_2Ereal\_mul \\ & V3x)\ (ap\ (ap\ c.2Epoly\_2Epoly\ V2t)\ V3x)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1p1 \in (ty\_2Elist\_2Elist \\ & ty\_2Erealax\_2Ereal).(\forall V2p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal). \\ & ((ap\ (ap\ c.2Epoly\_2Epoly\ (ap\ (ap\ c.2Epoly\_2Epoly\_mul\ V1p1)\ V2p2)) \\ & V0x) = (ap\ (ap\ c.2Erealax\_2Ereal\_mul\ (ap\ (ap\ c.2Epoly\_2Epoly\ V1p1) \\ & V0x))\ (ap\ (ap\ c.2Epoly\_2Epoly\ V2p2)\ V0x)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1a \in \\ & ty\_2Erealax\_2Ereal.(\neg((ap\ c.2Epoly\_2Epoly\ V0p) = (ap\ c.2Epoly\_2Epoly \\ & (c.2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))) \Rightarrow (p\ (ap\ (c.2Ebool\_2E3F\_21 \\ & ty\_2Enum\_2Enum)\ (\lambda V2n \in ty\_2Enum\_2Enum.(ap\ (ap\ c.2Ebool\_2E2F\_5C \\ & (ap\ (ap\ c.2Epoly\_2Epoly\_divides\ (ap\ (ap\ c.2Epoly\_2Epoly\_exp \\ & (ap\ (ap\ (c.2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c.2Erealax\_2Ereal\_neg \\ & V1a))\ (ap\ (ap\ (c.2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c.2Ereal\_2Ereal\_of\_num \\ & (ap\ c.2Earithmic\_2ENUMERAL\ (ap\ c.2Earithmic\_2EBIT1\ c.2Earithmic\_2EZERO)))) \\ & (c.2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ V2n))\ V0p))\ (ap\ c.2Ebool\_2E7E \\ & (ap\ (ap\ c.2Epoly\_2Epoly\_divides\ (ap\ (ap\ c.2Epoly\_2Epoly\_exp \\ & (ap\ (ap\ (c.2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c.2Erealax\_2Ereal\_neg \\ & V1a))\ (ap\ (ap\ (c.2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c.2Ereal\_2Ereal\_of\_num \\ & (ap\ c.2Earithmic\_2ENUMERAL\ (ap\ c.2Earithmic\_2EBIT1\ c.2Earithmic\_2EZERO)))) \\ & (c.2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ (ap\ c.2Enum\_2ESUC\ V2n))) \\ & V0p)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c.2Erealax\_2Ereal\_mul \\ & V0x)\ (ap\ c.2Ereal\_2Ereal\_of\_num\ c.2Enum\_2E0)) = (ap\ c.2Ereal\_2Ereal\_of\_num \\ & c.2Enum\_2E0))) \end{aligned} \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (64)$$



Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (65)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (69)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1a \in \\ & \quad ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.(((p (ap (ap \\ & \quad c\_2Epoly\_2Epoly\_divides (ap (ap c\_2Epoly\_2Epoly\_exp (ap (ap \\ & \quad (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\ & \quad V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\ & \quad (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) V2n)) V0p)) \wedge (\neg(p (ap \\ & \quad (ap c\_2Epoly\_2Epoly\_divides (ap (ap c\_2Epoly\_2Epoly\_exp (ap \\ & \quad (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\ & \quad V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\ & \quad (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) (ap c\_2Enum\_2ESUC V2n))) \\ & \quad V0p)))) \Leftrightarrow ((V2n = (ap (ap c\_2Epoly\_2Epoly\_order V1a) V0p)) \wedge (\neg( \\ & \quad (ap c\_2Epoly\_2Epoly V0p) = (ap c\_2Epoly\_2Epoly (c\_2Elist\_2ENIL \\ & \quad ty\_2Erealax\_2Ereal))))))))) \end{aligned}$$