

thm\_2Epoly\_2EORDER\_MUL  
(TMQkcX4dn2UL56X38LTR2MN9rUEr3WdLkHd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{6}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let  $c\_2Epoly\_2Epoly\_mul : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_mul \in ((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)} \quad (8)$$

Let  $c\_2Epoly\_2Epoly : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly \in ((ty\_2Erealax\_2Ereal)^{ty\_2Erealax\_2Ereal})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)} \quad (9)$$

**Definition 5** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 6** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E40\ A\_27a\ V0P))))$

**Definition 7** We define  $c\_2Epoly\_2Epoly\_divides$  to be  $\lambda V0p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (11)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2EZERO\ V0n))$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (13)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (14)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (16)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (t$   
Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (21) (17)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (18)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})} (19)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

Let  $c\_2Epoly\_2Epoly\_exp : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_exp \in (((ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal)^{ty\_2Eenum\_2Eenum}) (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal)) (20)$$

**Definition 15** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E.21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Emin\_2E.3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 17** We define  $c\_2Ebool\_2E.7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E.3D\_3D\_3E V0t) c\_2Ebool\_2E.7E$

**Definition 18** We define  $c\_2Ebool\_2E.2F.5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E.21 2) (\lambda V2t \in 2.V2t$

**Definition 19** We define  $c\_2Epoly\_2Epoly\_order$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.\lambda V1p \in (ty\_2Elist\_2E$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (21)$$

**Definition 20** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 21** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (22)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .  
Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))) \quad (23)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 24** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.))$   
Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow \neg(p\ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p\ V0t) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((\forall V2x \in \\
& A\_27a. ((ap \ V0f \ V2x) = (ap \ V1g \ V2x))) \Rightarrow (V0f = V1g))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \ V0A) \vee ( \\
& (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\
& (p \ V0A))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (43)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0f \in ((A_{27a}^{A_{27a}})^{A_{27a}}). \\ & ((\forall V1x \in A_{27a}.(\forall V2y \in A_{27a}.(\forall V3z \in A_{27a}. \\ & ((ap (ap V0f V1x) (ap (ap V0f V2y) V3z)) = (ap (ap V0f (ap (ap V0f V1x) \\ & V2y)) V3z)))))) \Rightarrow ((\forall V4x \in A_{27a}.(\forall V5y \in A_{27a}.((ap \\ & (ap V0f V4x) V5y) = (ap (ap V0f V5y) V4x)))) \Rightarrow (\forall V6x \in A_{27a}.( \\ & \forall V7y \in A_{27a}.(\forall V8z \in A_{27a}.((ap (ap V0f V6x) (ap (ap \\ & V0f V7y) V8z)) = (ap (ap V0f V7y) (ap (ap V0f V6x) V8z)))))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & ((\forall V0p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).((ap ( \\ & ap c\_2Epoly\_2Epoly\_exp V0p) c\_2Enum\_2E0) = (ap (ap (c\_2Elist\_2ECONS \\ & ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (c\_2Elist\_2ENIL \\ & ty\_2Erealax\_2Ereal)))) \wedge (\forall V1p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\ & (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Epoly\_2Epoly\_exp V1p) \\ & (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Epoly\_2Epoly\_mul V1p) (ap \\ & (ap c\_2Epoly\_2Epoly\_exp V1p) V2n)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1p1 \in (ty\_2Elist\_2Elist \\ & ty\_2Erealax\_2Ereal).(\forall V2p2 \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\ & ((ap (ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2Epoly\_mul V1p1) V2p2)) \\ & V0x) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Epoly\_2Epoly V1p1) \\ & V0x)) (ap (ap c\_2Epoly\_2Epoly V2p2) V0x)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0d \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & \forall V2p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).((ap c\_2Epoly\_2Epoly \\ & (ap (ap c\_2Epoly\_2Epoly\_exp V2p) (ap (ap c\_2Earithmetic\_2E\_2B \\ & V1n) V0d))) = (ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2Epoly\_mul \\ & (ap (ap c\_2Epoly\_2Epoly\_exp V2p) V1n)) (ap (ap c\_2Epoly\_2Epoly\_exp \\ & V2p) V0d)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1q \in \\
& (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(((ap\ c\_2Epoly\_2Epoly \\
& (ap\ (ap\ c\_2Epoly\_2Epoly\_mul\ V0p)\ V1q)) = (ap\ c\_2Epoly\_2Epoly\ ( \\
& c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))) \Leftrightarrow (((ap\ c\_2Epoly\_2Epoly \\
& V0p) = (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))) \vee \\
& ((ap\ c\_2Epoly\_2Epoly\ V1q) = (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1q \in \\
& (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V2r \in (ty\_2Elist\_2Elist \\
& ty\_2Erealax\_2Ereal).(((ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_mul \\
& V0p)\ V1q)) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_mul\ V0p) \\
& V2r))) \Leftrightarrow (((ap\ c\_2Epoly\_2Epoly\ V0p) = (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal))) \vee ((ap\ c\_2Epoly\_2Epoly\ V1q) = (ap\ c\_2Epoly\_2Epoly \\
& V2r))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg((ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_exp\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal)\ V0a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal) \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal)))) V1n)) = (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1p \in (ty\_2Elist\_2Elist \\
& ty\_2Erealax\_2Ereal). (\forall V2q \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\
& ((p (ap (ap c\_2Epoly\_2Epoly\_divides (ap (ap (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal) V0a) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal)))) (ap (ap c\_2Epoly\_2Epoly\_mul V1p) V2q))) \Leftrightarrow \\
& ((p (ap (ap c\_2Epoly\_2Epoly\_divides (ap (ap (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal) V0a) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal)))) V1p) \vee (p (ap (ap c\_2Epoly\_2Epoly\_divides \\
& (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) V0a) (ap (ap (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal)))))) V2q))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). (\forall V1a \in \\
& ty\_2Erealax\_2Ereal. (\forall V2n \in ty\_2Enum\_2Enum. (((p (ap (ap \\
& c\_2Epoly\_2Epoly\_divides (ap (ap c\_2Epoly\_2Epoly\_exp (ap (ap \\
& (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\
& V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) V2n)) V0p)) \wedge (\neg (p (ap \\
& (ap c\_2Epoly\_2Epoly\_divides (ap (ap c\_2Epoly\_2Epoly\_exp (ap \\
& (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\
& V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) (ap c\_2Enum\_2ESUC V2n))) \\
& V0p)))) \Leftrightarrow ((V2n = (ap (ap c\_2Epoly\_2Epoly\_order V1a) V0p)) \wedge (\neg ( \\
& (ap c\_2Epoly\_2Epoly V0p) = (ap c\_2Epoly\_2Epoly (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal))))))
\end{aligned} \tag{53}$$



Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal.((\neg((ap\ c\_2Epoly\_2Epoly\ V0p) = (ap\ c\_2Epoly\_2Epoly \\
& \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))) \Rightarrow ((p\ (ap\ (ap\ c\_2Epoly\_2Epoly\_divides \\
& \quad (ap\ (ap\ c\_2Epoly\_2Epoly\_exp\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal) \\
& \quad (ap\ c\_2Erealax\_2Ereal\_neg\ V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal) \\
& \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& \quad \quad ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (c\_2Elist\_2ENIL \\
& \quad \quad ty\_2Erealax\_2Ereal))))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_order\ V1a)\ V0p))) \\
& \quad V0p) \wedge (\neg(p\ (ap\ (ap\ c\_2Epoly\_2Epoly\_divides\ (ap\ (ap\ c\_2Epoly\_2Epoly\_exp \\
& \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ (ap\ c\_2Enum\_2ESUC\ (ap \\
& \quad (ap\ c\_2Epoly\_2Epoly\_order\ V1a)\ V0p))))\ V0p))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal.((\neg((ap\ c\_2Epoly\_2Epoly\ V0p) = (ap\ c\_2Epoly\_2Epoly \\
& \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))) \Rightarrow (\exists V2q \in (ty\_2Elist\_2Elist \\
& \quad ty\_2Erealax\_2Ereal).(((ap\ c\_2Epoly\_2Epoly\ V0p) = (ap\ c\_2Epoly\_2Epoly \\
& \quad (ap\ (ap\ c\_2Epoly\_2Epoly\_mul\ (ap\ (ap\ c\_2Epoly\_2Epoly\_exp\ (ap \\
& \quad (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_order \\
& \quad \quad V1a)\ V0p)))\ V2q))) \wedge (\neg(p\ (ap\ (ap\ c\_2Epoly\_2Epoly\_divides\ (ap\ ( \\
& \quad \quad ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Erealax\_2Ereal\_neg \\
& \quad V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \\
& \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ V2q))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_add \\
& \quad (ap\ c\_2Erealax\_2Ereal\_neg\ V0x)\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad \quad c\_2Enum\_2E0)))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\
& \quad \quad V1y)\ V0x))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V0x)))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x)))) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
V1y) = (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) V1y))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0y) V1x)))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Erealax\_2Ereal\_neg V0x)) \\
& (ap c\_2Erealax\_2Ereal\_neg V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1y) V0x))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Erealax\_2Ereal\_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) V1y)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{70}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (75)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (80)$$

### Theorem 1

$$\begin{aligned} & (\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1p \in (ty\_2Elist\_2Elist \\ & ty\_2Erealax\_2Ereal).(\forall V2q \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\ & ((\neg((ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2Epoly\_mul V1p) V2q)) = \\ & (ap c\_2Epoly\_2Epoly (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) \Rightarrow \\ & ((ap (ap c\_2Epoly\_2Epoly\_order V0a) (ap (ap c\_2Epoly\_2Epoly\_mul \\ & V1p) V2q)) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Epoly\_2Epoly\_order \\ & V0a) V1p)) (ap (ap c\_2Epoly\_2Epoly\_order V0a) V2q)))))) \end{aligned}$$