

# thm\_2Epoly\_2EORDER\_THM (TMZECX2wqgce25TZ2NmXLgaVbgKSvCWFhzA)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A))))$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Erealax\_2Ereal} \tag{1}$$

Let `ty_2Elist_2Elist` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty\_2Elist\_2Elist } A 0) \tag{2}$$

Let `c_2Epoly_2Epoly` :  $\iota$  be given. Assume the following.

$$\text{c\_2Epoly\_2Epoly} \in ((\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Erealax\_2Ereal}})^{\text{ty\_2Elist\_2Elist } \text{ty\_2Erealax\_2Ereal}}) \tag{3}$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Enum\_2Enum} \tag{4}$$

Let `c_2Enum_2EREP_num` :  $\iota$  be given. Assume the following.

$$\text{c\_2Enum\_2EREP\_num} \in (\text{omega}^{\text{ty\_2Enum\_2Enum}}) \tag{5}$$

Let `c_2Enum_2ESUC_REP` :  $\iota$  be given. Assume the following.

$$\text{c\_2Enum\_2ESUC\_REP} \in (\text{omega}^{\text{omega}}) \tag{6}$$

Let `c_2Enum_2EABS_num` :  $\iota$  be given. Assume the following.

$$\text{c\_2Enum\_2EABS\_num} \in (\text{ty\_2Enum\_2Enum}^{\text{omega}}) \tag{7}$$

**Definition 5** We define  $c\_Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_EEnum\_2ESUC$  to be  $\lambda V0m \in ty\_2EEnum\_2Enum. (ap c\_EEnum\_2EABS\_num ($

Let  $c\_Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (8)$$

Let  $c\_EEnum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_EEnum\_2EZERO\_REP \in omega \quad (9)$$

**Definition 7** We define  $c\_EEnum\_2E0$  to be  $(ap c\_EEnum\_2EABS\_num c\_EEnum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_Earithmic\_2EZERO$  to be  $c\_EEnum\_2E0$ .

Let  $c\_Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmic\_2E\_2B \in ((ty\_2EEnum\_2Enum^{ty\_2EEnum\_2Enum})^{ty\_2EEnum\_2Enum}) \quad (10)$$

**Definition 9** We define  $c\_Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2EEnum\_2Enum. (ap (ap c\_Earithmic\_2E\_2B$

**Definition 10** We define  $c\_Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2EEnum\_2Enum. V0x$ .

Let  $c\_Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2EEnum\_2Enum}) \quad (11)$$

Let  $c\_Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (12)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal \quad (13)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (14)$$

Let  $c\_Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (15)$$

**Definition 11** We define  $c\_Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap (c\_Emin\_2E\_40 ($



Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg (\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg (p (ap V0P V2x)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg (p V0A)) \vee (\neg (p V1B)))) \wedge ((\neg ((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg (p V0A)) \wedge (\neg (p V1B)))))) \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in (ty.2Elist.2Elist ty.2Erealax.2Ereal).(\forall V1a \in ty.2Erealax.2Ereal.(\forall V2n \in ty.2Enum.2Enum.(((p (ap c.2Epoly.2Epoly\_divides (ap (ap c.2Epoly.2Epoly\_exp (ap (ap c.2Elist.2ECONS ty.2Erealax.2Ereal) (ap c.2Erealax.2Ereal\_neg V1a)) (ap (ap (c.2Elist.2ECONS ty.2Erealax.2Ereal) (ap c.2Ereal.2Ereal\_of\_num (ap c.2Earithmic.2ENUMERAL (ap c.2Earithmic.2EBIT1 c.2Earithmic.2EZERO)))) (c.2Elist.2ENIL ty.2Erealax.2Ereal)))) V2n)) V0p)) \wedge (\neg (p (ap (ap c.2Epoly.2Epoly\_divides (ap (ap c.2Epoly.2Epoly\_exp (ap (ap c.2Elist.2ECONS ty.2Erealax.2Ereal) (ap c.2Erealax.2Ereal\_neg V1a)) (ap (ap (c.2Elist.2ECONS ty.2Erealax.2Ereal) (ap c.2Ereal.2Ereal\_of\_num (ap c.2Earithmic.2ENUMERAL (ap c.2Earithmic.2EBIT1 c.2Earithmic.2EZERO)))) (c.2Elist.2ENIL ty.2Erealax.2Ereal)))) (ap c.2Enum.2ESUC V2n))) V0p)))) \Leftrightarrow ((V2n = (ap (ap c.2Epoly.2Epoly\_order V1a) V0p)) \wedge (\neg (ap c.2Epoly.2Epoly V0p) = (ap c.2Epoly.2Epoly (c.2Elist.2ENIL ty.2Erealax.2Ereal)))))) \quad (39) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg (p V0A)) \Rightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (49)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1a \in \\ & \quad ty\_2Erealax\_2Ereal.((\neg((ap\ c\_2Epoly\_2Epoly\ V0p) = (ap\ c\_2Epoly\_2Epoly \\ & \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))) \Rightarrow ((p\ (ap\ (ap\ c\_2Epoly\_2Epoly\_divides \\ & \quad (ap\ (ap\ c\_2Epoly\_2Epoly\_exp\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal) \\ & \quad (ap\ c\_2Erealax\_2Ereal\_neg\ V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal) \\ & \quad (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\ & \quad ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ (c\_2Elist\_2ENIL \\ & \quad ty\_2Erealax\_2Ereal))))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_order\ V1a)\ V0p))) \\ & \quad V0p)) \wedge (\neg(p\ (ap\ (ap\ c\_2Epoly\_2Epoly\_divides\ (ap\ (ap\ c\_2Epoly\_2Epoly\_exp \\ & \quad (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Erealax\_2Ereal\_neg \\ & \quad V1a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ \\ & \quad (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ (ap\ c\_2Enum\_2ESUC\ (ap \\ & \quad (ap\ c\_2Epoly\_2Epoly\_order\ V1a)\ V0p))))\ V0p)))))) \end{aligned}$$