

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (5)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))} \quad (7)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)(ty_2Elist_2Elist\ A.27a))A.27a) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (10)$$

Definition 9 We define $c_2Ebool_2E_2F.5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Epoly_2Epoly_add : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_add \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)(ty_2Elist_2Elist\ ty_2Erealax_2Ereal))(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)) \quad (11)$$

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 11 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)) \Rightarrow (V0f = V1g)))))) \quad (22)$$

Assume the following.

$$((\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Epoly_2Epoly\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \wedge (\forall V1h \in ty_2Erealax_2Ereal.(\forall V2t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V3x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V1h)\ V2t))\ V3x) = (ap\ (ap\ c_2Erealax_2Ereal_add\ V1h)\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V3x)\ (ap\ (ap\ c_2Epoly_2Epoly\ V2t)\ V3x)))))))))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p1 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1p2 \in \\
& (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V2x \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ c_2Epoly_2Epoly_add\ V0p1)\ V1p2)) \\
& V2x) = (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap\ c_2Epoly_2Epoly\ V0p1) \\
& V2x))\ (ap\ (ap\ c_2Epoly_2Epoly\ V1p2)\ V2x))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_add \\
& V0x)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = V0x))
\end{aligned} \tag{25}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).((ap\ c_2Epoly_2Epoly \\
& (ap\ (ap\ c_2Epoly_2Epoly_add\ V0p)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) = \\
& (ap\ c_2Epoly_2Epoly\ V0p))
\end{aligned}$$