

thm_2Epoly_2EPOLY__CMUL__CLAUSES (TMW7NiJM8D5Wghqd5Rba9QsjFG77qztPQAn)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A \cdot 27a}).(ap (ap (c_2Emin_2E_3D (2^{A \cdot 27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Etrealmul) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (10)$$

Let $c_2Epoly_2E23_23 : \iota$ be given. Assume the following.

$$c_2Epoly_2E23_23 \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{ty_2Erealax_2Ereal}) \quad (11)$$

Definition 9 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.))$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0c \in ty_2Erealax_2Ereal. ((ap (ap c_2Epoly_2E_23_23 \\
& V0c) (c_2Elist_2ENIL ty_2Erealax_2Ereal)) = (c_2Elist_2ENIL \\
& ty_2Erealax_2Ereal))) \wedge (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2h \in \\
& ty_2Erealax_2Ereal. (\forall V3t \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). \\
& ((ap (ap c_2Epoly_2E_23_23 V1c) (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) \\
& V2h) V3t)) = (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) (ap (\\
& ap c_2Erealax_2Ereal_mul V1c) V2h)) (ap (ap c_2Epoly_2E_23_23 \\
& V1c) V3t))))))))))
\end{aligned} \tag{15}$$

Theorem 1

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1h \in ty_2Erealax_2Ereal. \\
& (\forall V2t \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (((ap (\\
& ap c_2Epoly_2E_23_23 V0c) (c_2Elist_2ENIL ty_2Erealax_2Ereal)) = \\
& (c_2Elist_2ENIL ty_2Erealax_2Ereal)) \wedge ((ap (ap c_2Epoly_2E_23_23 \\
& V0c) (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) V1h) V2t)) = \\
& (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) (ap (ap c_2Erealax_2Ereal_mul \\
& V0c) V1h)) (ap (ap c_2Epoly_2E_23_23 V0c) V2t))))))))))
\end{aligned}$$