

thm\_2Epoly\_2EPOLY\_\_DIFF  
(TMQuJW9nv1YPpY4LRcJTRSpSwL3b2W9pEGA)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))^{A\_27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (3)$$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealx\_2Ereal \quad (4)$$

Let  $c\_2Elist\_2ETL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ETL A\_27a \in ((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a)) \quad (5)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (6)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap \ c\_2Enum\_2EABS\_num \ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap \ c\_2Enum\_2EABS\_num \ m)$

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 10** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap \ (ap \ c\_2Earithmic\_2E\_2B \ n)) \ V0n)$

**Definition 11** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Epoly\_2Epoly\_diff\_aux : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_diff\_aux \in (((ty\_2Elist\_2Elist \ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist \ ty\_2Erealax\_2Ereal)})^{ty\_2Elist\_2Elist \ ty\_2Erealax\_2Ereal}) \quad (12)$$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap \ (c\_2Ebool\_2E\_21 \ 2) \ (\lambda V2t \in 2. (c\_2Ebool\_2E\_21 \ t2) \ t2) \ t1)) \ V0t1)$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then } (the \ (\lambda x. x \in A \ \wedge \ P \ x)) \ \text{of type } \iota \Rightarrow \iota.$

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. \lambda V2t2 \in A. \lambda V3t3 \in A. \lambda V4t4 \in A. (c\_2Ebool\_2E\_21 \ t4) \ (c\_2Ebool\_2E\_21 \ t3) \ t2) \ t1) \ V0t)$

**Definition 15** We define  $c\_2Epoly\_2Ediff$  to be  $\lambda V0l \in (ty\_2Elist\_2Elist \ ty\_2Erealax\_2Ereal). (ap \ (ap \ (ap \ (ap \ (ap \ (c\_2Epoly\_2Epoly\_diff\_aux \ l) \ c) \ c) \ c) \ c) \ c)$

Let  $c\_2Epoly\_2Epoly : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)}) \quad (13)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (14)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (15)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (16)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (17)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ t$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (18)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (20)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (21)$$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 20** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (22)$$

**Definition 21** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (23)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 23** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (24)$$

**Definition 24** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 25** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 26** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECONJ$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (25)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (26)$$

**Definition 27** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a \times A\_27b})$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (27)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}}) \quad (28)$$

**Definition 28** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal) (ap (c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b}})^{A\_27a}) \quad (29)$$

**Definition 29** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Emetric\ A\_27a\ A\_27a))}) \quad (30)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(2^{A\_27a})}) \quad (31)$$

**Definition 30** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap (c\_2Emin\_2E40$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (32)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (33)$$

**Definition 31** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}} \quad (34)$$

**Definition 32** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

**Definition 33** We define  $c\_2Elim\_2Ediff1$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum})^{ty\_2Erealax\_2Ereal}) \quad (35)$$

Assume the following.

$$((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0)) \quad (36)$$

Assume the following.

$$True \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27b. ((ap\ (\lambda V2x \in A\_27b. \\ V0t1)\ V1t2) = V0t1))) \end{aligned} \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (39)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ p\ V0t)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0k \in ty\_2Erealax\_2Ereal. (\forall V1x \in ty\_2Erealax\_2Ereal. \\ (p\ (ap\ (ap\ (ap\ c\_2Elim\_2Ediff\ (\lambda V2x \in ty\_2Erealax\_2Ereal.V0k)) \\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V1x)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V1g \in \\ (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). (\forall V2l \in ty\_2Erealax\_2Ereal. \\ (\forall V3m \in ty\_2Erealax\_2Ereal. (\forall V4x \in ty\_2Erealax\_2Ereal. \\ (((p\ (ap\ (ap\ (ap\ c\_2Elim\_2Ediff\ V0f)\ V2l)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ c\_2Elim\_2Ediff \\ V1g)\ V3m)\ V4x))) \Rightarrow (p\ (ap\ (ap\ (ap\ c\_2Elim\_2Ediff\ (\lambda V5x \in ty\_2Erealax\_2Ereal. \\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap \\ (ap\ c\_2Erealax\_2Ereal\_add\ V2l)\ V3m))\ V4x)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ A\_27a). (p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Epoly\_2Epoly ( \\
& c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \wedge (\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in \\
& (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(\forall V3x \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Epoly\_2Epoly (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) \\
& V1h) V2t)) V3x) = (ap (ap c\_2Erealax\_2Ereal\_add V1h) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V3x) (ap (ap c\_2Epoly\_2Epoly V2t) V3x))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0c \in ty\_2Erealax\_2Ereal.(\forall V1h \in ty\_2Erealax\_2Ereal. \\
& (\forall V2t \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(((ap c\_2Epoly\_2Ediff \\
& (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)) \wedge \\
& (((ap c\_2Epoly\_2Ediff (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) \\
& V0c) (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal))) = (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal)) \wedge ((ap c\_2Epoly\_2Ediff (ap (ap (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal) V1h) V2t)) = (ap (ap c\_2Epoly\_2Epoly\_diff\_aux \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& V2t)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0l \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(\forall V1n \in \\
& ty\_2Enum\_2Enum.(\forall V2x \in ty\_2Erealax\_2Ereal.(p (ap (ap ( \\
& ap c\_2Elim\_2Ediff1 (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Ereal\_2Epow V3x) (ap c\_2Enum\_2ESUC V1n))) (ap (ap c\_2Epoly\_2Epoly \\
& V0l) V3x)))) (ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Ereal\_2Epow \\
& V2x) V1n)) (ap (ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2Epoly\_diff\_aux \\
& (ap c\_2Enum\_2ESUC V1n)) V0l)) V2x))) V2x))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = V0x))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Ereal\_2Epow V0x) \\
c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Ereal\_2Epow \\
& V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Ereal\_2Epow V0x) \\
(ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
& V0x))
\end{aligned} \tag{51}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0l \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(\forall V1x \in \\
ty\_2Erealax\_2Ereal.(p (ap (ap (ap c\_2Elim\_2Ediff1 (\lambda V2x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap c\_2Epoly\_2Epoly V0l) V2x))) (ap (ap c\_2Epoly\_2Epoly (ap \\
& c\_2Epoly\_2Ediff V0l)) V1x)) V1x))))
\end{aligned}$$