thm_2Epoly_2EPOLY__DIFF__AUX__CMUL (TMQE7hQaD71dirF77Atr1dmxoDNXYatdVz5)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$ *of type* ι.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 4 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (5)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (6)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \quad (7)$$

Definition 7 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 8 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (8)$$

Let $c\_2Epoly\_2Epoly : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)}) \quad (9)$$

Let $c\_2Epoly\_2E\_23\_23 : \iota$ be given. Assume the following.

$$c\_2Epoly\_2E\_23\_23 \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{ty\_2Ereal} \quad (10)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (13)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (14)$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{15}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{16}$$

Let $c\_2Epoly\_2Epoly\_\_diff\_\_aux : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_diff\_\_aux \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)} \tag{17}$$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{18}$$

Definition 10 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{19}$$

Definition 11 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{20}$$

Definition 12 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{21}$$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x))) \Rightarrow (V0f = V1g)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}). ((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a. ((ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z)) = (ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x)\ V2y))\ V3z))))) \Rightarrow ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((ap\ (ap\ V0f\ V4x)\ V5y) = (ap\ (ap\ V0f\ V5y)\ V4x)))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.(\forall V8z \in A\_27a.((ap\ (ap\ V0f\ V6x)\ (ap\ (ap\ V0f\ V7y)\ V8z)) = (ap\ (ap\ V0f\ V7y)\ (ap\ (ap\ V0f\ V6x)\ V8z))))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{28}$$

Assume the following.

$$((\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \wedge (\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V3x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ V3x) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1h)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V3x)\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V2t)\ V3x)))))))) \tag{29}$$

Assume the following.

$$((\forall V0c \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V0c)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\wedge(\forall V1c \in ty\_2Erealax\_2Ereal.(\forall V2h \in ty\_2Erealax\_2Ereal.(\forall V3t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1c)\ V2h))\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ V3t)))))))) \tag{30}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ V0n)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\wedge(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2h \in ty\_2Erealax\_2Ereal.(\forall V3t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ V1n)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1n))\ V2h))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V3t)))))))) \tag{31}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V0x)))) \tag{32}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))\ V2z))))) \tag{33}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((((ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m))\Leftrightarrow(V0n = V1m))\wedge((((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m))\Leftrightarrow((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge((((ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m)))\Leftrightarrow((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge(((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n)) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V1m)))\Leftrightarrow(V0n = V1m))))))) \tag{34}$$

Theorem 1

$$(\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1c \in ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ V2n)\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ V0p))) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ V2n)\ V0p)))))))$$