

thm_2Epoly_2EPOLY__DIFF__AUX__ISZERO (TMSzmMK7EBPtjLRVHioUehRZpv1aR7Efjny)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2EEVERY` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \text{c_2Elist_2EEVERY } A_27a \in ((2^{(\text{ty_2Elist_2Elist } A_27a)})^{(2^{A_27a})}) \quad (2)$$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF}))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \quad (3)$$

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EREP_num} \in (\text{omega}^{\text{ty_2Enum_2Enum}}) \quad (4)$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$\text{c_2Enum_2ESUC_REP} \in (\text{omega}^{\text{omega}}) \quad (5)$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EABS_num} \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \quad (6)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (8)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (9)$$

Let $c_2Epoly_2Epoly_diff_aux : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_diff_aux \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{ty_2Erealax_2Ereal}) \quad (10)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (13)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}) \quad (14)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (the\ (\lambda x.x \in A \wedge P\ x))))$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (15)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (16)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (17)$$

Definition 13 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 14 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum} \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((p (ap \\
& (ap (c.2Elist.2EVERY A.27a) V0P) (c.2Elist.2ENIL A.27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((p (ap (ap (c.2Elist.2EVERY A.27a) V1P) (ap (ap (c.2Elist.2ECONS \\
& A.27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \wedge (p (ap (ap (c.2Elist.2EVERY \\
& A.27a) V1P) V3t))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist A.27a)}). \\
& (((p (ap V0P (c.2Elist.2ENIL A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a.(p (ap V0P (ap (ap (\\
& c.2Elist.2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p (ap V0P V3l))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0n \in ty.2Enum.2Enum.(\neg((ap c.2Enum.2ESUC V0n) = c.2Enum.2E0))) \tag{28}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty.2Enum.2Enum.((ap (ap c.2Epoly.2Epoly_diff_aux \\
& V0n) (c.2Elist.2ENIL ty.2Erealax.2Ereal)) = (c.2Elist.2ENIL \\
& ty.2Erealax.2Ereal))) \wedge (\forall V1n \in ty.2Enum.2Enum.(\forall V2h \in \\
& ty.2Erealax.2Ereal.(\forall V3t \in (ty.2Elist.2Elist ty.2Erealax.2Ereal). \\
& ((ap (ap c.2Epoly.2Epoly_diff_aux V1n) (ap (ap (c.2Elist.2ECONS \\
& ty.2Erealax.2Ereal) V2h) V3t)) = (ap (ap (c.2Elist.2ECONS ty.2Erealax.2Ereal) \\
& (ap (ap c.2Erealax.2Ereal_mul (ap c.2Ereal.2Ereal_of_num \\
& V1n)) V2h)) (ap (ap c.2Epoly.2Epoly_diff_aux (ap c.2Enum.2ESUC \\
& V1n)) V3t)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealax.2Ereal.(\forall V1y \in ty.2Erealax.2Ereal. \\
& (((ap (ap c.2Erealax.2Ereal_mul V0x) V1y) = (ap c.2Ereal.2Ereal_of_num \\
& c.2Enum.2E0)) \Leftrightarrow ((V0x = (ap c.2Ereal.2Ereal_of_num c.2Enum.2E0)) \vee \\
& (V1y = (ap c.2Ereal.2Ereal_of_num c.2Enum.2E0))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Ereal_2Ereal_of_num\ V0m) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \\
& \hspace{20em} (31)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (\forall V1n \in \\
& \quad ty_2Enum_2Enum. ((p\ (ap\ (ap\ (c_2Elist_2EVERY\ ty_2Erealax_2Ereal) \\
& \quad (\lambda V2c \in ty_2Erealax_2Ereal. (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Erealax_2Ereal) \\
V2c)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))\ (ap\ (ap\ c_2Epoly_2Epoly_diff_aux \\
& \quad (ap\ c_2Enum_2ESUC\ V1n)\ V0p))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Elist_2EVERY\ ty_2Erealax_2Ereal) \\
& \quad (\lambda V3c \in ty_2Erealax_2Ereal. (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Erealax_2Ereal) \\
& \quad V3c)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))\ V0p))))))
\end{aligned}$$