

thm\_2Epoly\_2EPOLY\_\_DIFF\_\_EXP\_\_PRIME  
(TMK5oBR1mbgTZczznMHWWktVSKnNBLZAjYA)

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Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (4)$$

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p\ (ap\ P\ x))$  **then** (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a})))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ (ty\_2Erealax\_2Ereal\_REP\_CLASS\ a)))$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (5)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg)$

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 9** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (8)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (9)$$

**Definition 11** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Elist\_2ETL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (10)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let  $c\_2Epoly\_2Epoly\_diff\_aux : \iota$  be given. Assume the following.

$$c\_2Epoly\_2Epoly\_diff\_aux \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)}) \quad (12)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (13)$$



**Definition 19** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ . Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (24)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (25)$$

**Definition 20** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ . Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((\forall V2x \in A\_27a.((ap V0f V2x) = (ap V1g V2x)) \Rightarrow (V0f = V1g)))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}). \\ & ((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a. \\ & ((ap (ap V0f V1x) (ap (ap V0f V2y) V3z)) = (ap (ap V0f (ap (ap V0f V1x) \\ & V2y)) V3z)))))) \Rightarrow ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((ap \\ & (ap V0f V4x) V5y) = (ap (ap V0f V5y) V4x)))) \Rightarrow (\forall V6x \in A\_27a.( \\ & \forall V7y \in A\_27a.(\forall V8z \in A\_27a.((ap (ap V0f V6x) (ap (ap \\ & V0f V7y) V8z)) = (ap (ap V0f V7y) (ap (ap V0f V6x) V8z))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in \\ & (ty\_2Elist\_2Elist A\_27a).((ap (c\_2Elist\_2ETL A\_27a) (ap (ap ( \\ & c\_2Elist\_2ECONS A\_27a) V0h) V1t)) = V1t))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist \\ & A\_27a).(\forall V1a0 \in A\_27a.(\neg((ap (ap (c\_2Elist\_2ECONS A\_27a) \\ & V1a0) V0a1) = (c\_2Elist\_2ENIL A\_27a)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & ((\forall V0x \in ty\_2Erealx\_2Ereal.((ap (ap c\_2Epoly\_2Epoly ( \\ & c\_2Elist\_2ENIL ty\_2Erealx\_2Ereal)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0))) \wedge (\forall V1h \in ty\_2Erealx\_2Ereal.(\forall V2t \in \\ & (ty\_2Elist\_2Elist ty\_2Erealx\_2Ereal).(\forall V3x \in ty\_2Erealx\_2Ereal. \\ & ((ap (ap c\_2Epoly\_2Epoly (ap (ap (c\_2Elist\_2ECONS ty\_2Erealx\_2Ereal) \\ & V1h) V2t)) V3x) = (ap (ap c\_2Erealx\_2Ereal\_add V1h) (ap (ap c\_2Erealx\_2Ereal\_mul \\ & V3x) (ap (ap c\_2Epoly\_2Epoly V2t) V3x))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Epoly\_2Epoly\_diff\_aux \\
& V0n) (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL \\
& ty\_2Erealax\_2Ereal))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2h \in \\
& ty\_2Erealax\_2Ereal.(\forall V3t \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\
& ((ap (ap c\_2Epoly\_2Epoly\_diff\_aux V1n) (ap (ap (c\_2Elist\_2ECONS \\
& ty\_2Erealax\_2Ereal) V2h) V3t)) = (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) \\
& (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1n)) V2h)) (ap (ap c\_2Epoly\_2Epoly\_diff\_aux (ap c\_2Enum\_2ESUC \\
& V1n)) V3t))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(\forall V1c \in \\
& ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((ap ( \\
& ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2E\_23\_23 V1c) V0p)) V2x) = ( \\
& ap (ap c\_2Erealax\_2Ereal\_mul V1c) (ap (ap c\_2Epoly\_2Epoly V0p) \\
& V2x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1p1 \in (ty\_2Elist\_2Elist \\
& ty\_2Erealax\_2Ereal).(\forall V2p2 \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal). \\
& ((ap (ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2Epoly\_mul V1p1) V2p2)) \\
& V0x) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Epoly\_2Epoly V1p1) \\
& V0x)) (ap (ap c\_2Epoly\_2Epoly V2p2) V0x))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty\_2Elist\_2Elist ty\_2Erealax\_2Ereal).(\forall V1n \in \\
& ty\_2Enum\_2Enum.((ap c\_2Epoly\_2Epoly (ap c\_2Epoly\_2Ediff (ap \\
& (ap c\_2Epoly\_2Epoly\_exp V0p) (ap c\_2Enum\_2ESUC V1n)))) = (ap c\_2Epoly\_2Epoly \\
& (ap (ap c\_2Epoly\_2Epoly\_mul (ap (ap c\_2Epoly\_2E\_23\_23 (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Enum\_2ESUC V1n))) (ap (ap c\_2Epoly\_2Epoly\_exp V0p) V1n))) \\
& (ap c\_2Epoly\_2Ediff V0p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V0x))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{48}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Erealax\_2Ereal. \\
& ((ap c\_2Epoly\_2Epoly (ap c\_2Epoly\_2Ediff (ap (ap c\_2Epoly\_2Epoly\_exp \\
& (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\
V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) (ap c\_2Enum\_2ESUC V0n)))) = \\
& (ap c\_2Epoly\_2Epoly (ap (ap c\_2Epoly\_2E\_23\_23 (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Enum\_2ESUC V0n)) (ap (ap c\_2Epoly\_2Epoly\_exp (ap (ap ( \\
& c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Erealax\_2Ereal\_neg \\
V1a)) (ap (ap (c\_2Elist\_2ECONS ty\_2Erealax\_2Ereal) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& (c\_2Elist\_2ENIL ty\_2Erealax\_2Ereal)))) V0n))))))
\end{aligned}$$