

thm_2Epoly_2EPOLY__DIFF__LEMMA (TMc- trW8pxT9hEJ9qVy2QD3eWHxkWBbJ3hrm)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{5}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (6)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (7)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (9)$$

Definition 8 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$.($ap\ (c_2Emin_2E40\ (ty$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (12)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 12 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$.($ap\ c_2Erealax_2Ereal$

Definition 13 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal$. $\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (14)$$

Definition 14 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (15)$$

Definition 15 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 16 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \end{aligned} \quad (16)$$

Definition 17 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 18 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 19 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 21 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (17)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (18)$$

Definition 22 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (19)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}}) \end{aligned} \quad (20)$$

Definition 23 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (21)$$

Definition 24 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Enets_2Etendsto A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod (ty_2Emetric_2Emetric A_27a A_27b))}) \quad (22)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod A_27a A_27b)})^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 25 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (24)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (25)$$

Definition 26 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a). (ap (c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a)^{A_27b}}) \quad (26)$$

Definition 27 We define $c_2Elim_2Etends_real_real$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal}).$

Definition 28 We define $c_2Elim_2Ediff1$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal}). \lambda V1l \in ty_2Erealx_2Ereal.$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (27)$$

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal})^{(ty_2Elist_2Elist ty_2Erealx_2Ereal)}) \quad (28)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (29)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (30)$$

Let $c_2Epoly_2Epoly_diff_aux : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_diff_aux \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{ty_2Erealax_2Ereal}) \quad (31)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (32)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (33)$$

Definition 29 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 30 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (34)$$

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 32 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = V0m)) \quad (36)$$

Assume the following.

$$True \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))))) \quad (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& p V0t))))))))) \quad (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (ap (ap c.2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal.V0k)) \\
& (ap c.2Ereal_2Ereal_of_num c.2Enum_2E0)) V1x)))))) \quad (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap (ap c.2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c.2Elim_2Ediff1 \\
& V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c.2Erealax_2Ereal_add (ap V0f V5x)) (ap V1g V5x))) (ap \\
& (ap c.2Erealax_2Ereal_add V2l) V3m)) V4x))))))))) \quad (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1c \in \\
& ty_2Erealax_2Ereal.(\forall V2l \in ty_2Erealax_2Ereal.(\forall V3x \in \\
& ty_2Erealax_2Ereal.((p (ap (ap (ap c.2Elim_2Ediff1 V0f) V2l) V3x)) \Rightarrow \\
& (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V4x \in ty_2Erealax_2Ereal.(\\
& ap (ap c.2Erealax_2Ereal_mul V1c) (ap V0f V4x)))) (ap (ap c.2Erealax_2Ereal_mul \\
& V1c) V2l)) V3x))))))))) \quad (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal.(\\
& ap (ap c.2Ereal_2Epow V2x) V0n))) (ap (ap c.2Erealax_2Ereal_mul \\
& (ap c.2Ereal_2Ereal_of_num V0n)) (ap (ap c.2Ereal_2Epow V1x) \\
& (ap (ap c.2Earithmetic_2E_2D V0n) (ap c.2Earithmetic_2ENUMERAL \\
& (ap c.2Earithmetic_2EBIT1 c.2Earithmetic_2EZERO)))))) V1x)))) \quad (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& \quad c_2Elist_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Epoly_2Epoly\ (\\
& \quad c_2Elist_2ENIL\ ty_2Erealax_2Ereal))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0))) \wedge (\forall V1h \in ty_2Erealax_2Ereal.(\forall V2t \in \\
& \quad (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V3x \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
& \quad V1h)\ V2t))\ V3x) = (ap\ (ap\ c_2Erealax_2Ereal_add\ V1h)\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad V3x)\ (ap\ (ap\ c_2Epoly_2Epoly\ V2t)\ V3x)))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Epoly_2Epoly_diff_aux \\
& \quad V0n)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) = (c_2Elist_2ENIL \\
& \quad \quad ty_2Erealax_2Ereal))) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2h \in \\
& \quad ty_2Erealax_2Ereal.(\forall V3t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\
& \quad ((ap\ (ap\ c_2Epoly_2Epoly_diff_aux\ V1n)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad ty_2Erealax_2Ereal)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
& \quad \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad V1n))\ V2h))\ (ap\ (ap\ c_2Epoly_2Epoly_diff_aux\ (ap\ c_2Enum_2ESUC \\
& \quad \quad V1n))\ V3t))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_add \\
& \quad \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y))\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad \quad V0x)\ V2z))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& \quad \quad V1y)\ V0x))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Epow V0x) \\
c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge (\forall V1x \in \\
ty_2Erealax_2Ereal. (\forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Ereal_2Epow \\
V1x) (ap c_2Enum_2ESUC V2n)) = (ap (ap c_2Erealax_2Ereal_mul V1x) \\
& (ap (ap c_2Ereal_2Epow V1x) V2n))))))
\end{aligned} \tag{53}$$

Theorem 1

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V1n \in \\
ty_2Enum_2Enum. (\forall V2x \in ty_2Erealax_2Ereal. (p (ap (ap (\\
ap c_2Elim_2Ediff1 (\lambda V3x \in ty_2Erealax_2Ereal. (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2Epow V3x) (ap c_2Enum_2ESUC V1n))) (ap (ap c_2Epoly_2Epoly \\
V0l) V3x)))) (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Ereal_2Epow \\
V2x) V1n)) (ap (ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_diff_aux \\
& (ap c_2Enum_2ESUC V1n)) V0l)) V2x))) V2x))))))
\end{aligned}$$