thm_2Epoly_2EPOLY__DIFF__MUL (TMKPGc9vCnhef4JiCoLbRu89iX3j58vu8AN)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Let $c\_2Elist\_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{5}$$

Let $c\_2Epoly\_2Epoly\_\_diff\_\_aux : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_diff\_\_aux \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)} \tag{6}$$

Let $c\_2Epoly\_2Epoly\_\_mul : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_mul \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2I} \tag{7}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{8}$$

Let $c\_2Epoly\_2E\_23\_23 : \iota$ be given. Assume the following.

$$c\_2Epoly\_2E\_23\_23 \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{ty\_2Ereal} \tag{9}$$

Let $c\_2Epoly\_2Epoly\_\_add : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_add \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2E} \tag{10}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{11}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{13}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 10 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{15}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 12 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 14 *We define* c_2Epoly_2Ediff *to be* $\lambda V0l \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(ap\ (ap\ (ap$

Let $c\_2Epoly\_2Epoly : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)}) \tag{17}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{18}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{19}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{20}$$

Definition 15 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ ($c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{21}$$

Definition 16 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{22}$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{23}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{24}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{25}$$

Definition 17 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 18 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_neg}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal}$

Definition 19 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 20 *We define* $\mathsf{c\_2Ereal\_2Ereal\_\_lte}$ *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{26}$$

Definition 21 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_add}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{27}$$

Definition 22 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_mul}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 23 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{34}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x))) \Rightarrow (V0f = V1g)))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{42}$$

Assume the following.

$$((\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Epoly\_2Epoly\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\wedge(\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V3x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ V3x) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1h)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V3x)\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V2t)\ V3x))))))))) \tag{43}$$

Assume the following.

$$((\forall V0l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0l2) = V0l2))\wedge(\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V3l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ V3l2)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1h)\ (ap\ (c\_2Elist\_2EHD\ ty\_2Erealax\_2Ereal)\ V3l2)))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V2t)\ (ap\ (c\_2Elist\_2ETL\ ty\_2Erealax\_2Ereal)\ V3l2)))))))) \tag{44}$$

Assume the following.

$$((\forall V0l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0l2) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\wedge(\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal). (\forall V3l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ V2t)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1h)\ V3l2))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1h)\ V3l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ V2t)\ V3l2)))))))) \tag{45}$$

Assume the following.

$$(\forall V0c \in ty\_2Erealax\_2Ereal.(\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(((ap\ \mathsf{c\_2Epoly\_2Ediff}\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) \wedge (((ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V0c)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) \wedge ((ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t)) = (ap\ (ap\ c\_2Epoly\_2Epoly\_\_diff\_\_aux\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V2t))))))) \tag{46}$$

Assume the following.

$$(\forall V0p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V2x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V0p1)\ V1p2))\ V2x) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V0p1)\ V2x))\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V1p2)\ V2x)))))) \tag{47}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V2p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ V1p1)\ V2p2))\ V0x) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V1p1)\ V0x))\ (ap\ (ap\ c\_2Epoly\_2Epoly\ V2p2)\ V0x)))))) \tag{48}$$

Assume the following.

$$(\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V0p)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))) = (ap\ c\_2Epoly\_2Epoly\ V0p))) \tag{49}$$

Assume the following.

$$(\forall V0p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ c\_2Epoly\_2Epoly\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V0p1)\ V1p2))) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V0p1))\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V1p2)))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1c \in ty\_2Erealax\_2Ereal.((ap\ c\_2Epoly\_2Epoly\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ V0p))) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V0p)))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1h \in ty\_2Erealax\_2Ereal.((ap\ c\_2Epoly\_2Epoly\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V0t))) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V0t)))\ V0t)))))\tag{52}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V0x))))\tag{53}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z)))))\tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = V0x))\tag{55}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\tag{56}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V0x))))\tag{57}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))\ V2z)))))\tag{58}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x))\tag{59}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \tag{60}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{61}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))))) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{63}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V0x))) \tag{64}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x)))\Leftrightarrow(V0x = V1y)))) \tag{65}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))))) \tag{66}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V1x)\ V0y))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ V1x)))))) \tag{67}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)))))) \tag{68}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x))))) \tag{69}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = V0x)) \tag{70}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))) \tag{71}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z)))))) \tag{72}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z)))))) \tag{73}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{77}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{80}$$

Theorem 1

$$(\forall V0p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ c\_2Epoly\_2Epoly\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ V0p1)\ V1p2))) = (ap\ c\_2Epoly\_2Epoly\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ V0p1)\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V1p2)))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (ap\ \mathsf{c\_2Epoly\_2Ediff}\ V0p1))\ V1p2))))))$$