

thm_2Epoly_2EPOLY__ENTIRE (TMcRoLQvko7S2SC8HV78xiiLkpq4rin2upe)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})_{ty_2Erealax}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ECONS\ A.27a \in (((ty_2Elist_2Elist\ A.27a)^{(ty_2Elist_2Elist\ A.27a)})^{A.27a}) \quad (9)$$

Let $c_2Epoly_2Epoly_mul : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_mul \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)}) \quad (10)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (11)$$

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)}) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (16)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (17)$$

Definition 9 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 10 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))$

Definition 13 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))$

Definition 14 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).(\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)) \Rightarrow (V0f = V1g)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (31)$$

Assume the following.

$$((\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Epoly_2Epoly (c_2Elist_2ENIL ty_2Erealax_2Ereal)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \wedge (\forall V1h \in ty_2Erealax_2Ereal.(\forall V2t \in (ty_2Elist_2Elist ty_2Erealax_2Ereal).(\forall V3x \in ty_2Erealax_2Ereal.((ap (ap c_2Epoly_2Epoly (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) V1h) V2t)) V3x) = (ap (ap c_2Erealax_2Ereal_add V1h) (ap (ap c_2Erealax_2Ereal_mul V3x) (ap (ap c_2Epoly_2Epoly V2t) V3x)))))))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1p1 \in (ty_2Elist_2Elist \\
& ty_2Erealax_2Ereal). (\forall V2p2 \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). \\
& ((ap (ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_mul V1p1) V2p2)) \\
& V0x) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Epoly_2Epoly V1p1) \\
& V0x)) (ap (ap c_2Epoly_2Epoly V2p2) V0x))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V1q \in \\
& (ty_2Elist_2Elist ty_2Erealax_2Ereal). (((\neg((ap c_2Epoly_2Epoly \\
& V0p) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL ty_2Erealax_2Ereal)))) \wedge \\
& (\neg((ap c_2Epoly_2Epoly V1q) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL \\
& ty_2Erealax_2Ereal)))))) \Rightarrow (\neg((ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_mul \\
& V0p) V1q)) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL ty_2Erealax_2Ereal))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{46}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (\forall V1q \in \\
& (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (((ap\ c_2Epoly_2Epoly \\
& (ap\ (ap\ c_2Epoly_2Epoly_mul\ V0p)\ V1q)) = (ap\ c_2Epoly_2Epoly\ (\\
& c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \Leftrightarrow (((ap\ c_2Epoly_2Epoly \\
& V0p) = (ap\ c_2Epoly_2Epoly\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \vee \\
& ((ap\ c_2Epoly_2Epoly\ V1q) = (ap\ c_2Epoly_2Epoly\ (c_2Elist_2ENIL \\
& ty_2Erealax_2Ereal))))))
\end{aligned}$$