thm_2Epoly_2EPOLY__LENGTH__MUL (TMXxCqdWyiryncG7iR8PKttw4rLkwt5WgZH)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{9}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{10}$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{11}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{12}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{13}$$

Definition 11 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 12 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 13 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{14}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{15}$$

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (16)$$

Definition 16 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (17)$$

Let $c\_2Epoly\_2E\_23\_23 : \iota$ be given. Assume the following.

$$c\_2Epoly\_2E\_23\_23 \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{ty\_2Ereal} \quad (18)$$

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Epoly\_2Epoly\_\_mul : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_mul \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2E} \quad (19)$$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (20)$$

Definition 19 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (21)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (22)$$

Let $c\_2Epoly\_2Epoly\_\_add : \iota$ be given. Assume the following.

$$c\_2Epoly\_2Epoly\_\_add \in (((ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)^{(ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal)})^{(ty\_2E} \quad (23)$$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = \mathsf{c\_2Enum\_2E0}) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ A\_27a)\ V1t)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1a0 \in A\_27a.(\neg((ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a0)\ V0a1) = (c\_2Elist\_2ENIL\ A\_27a))))) \tag{32}$$

Assume the following.

$$((\forall V0c \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V0c)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\wedge(\forall V1c \in ty\_2Erealax\_2Ereal.(\forall V2h \in ty\_2Erealax\_2Ereal.(\forall V3t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1c)\ V2h))\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1c)\ V3t)))))))) \tag{33}$$

Assume the following.

$$((\forall V0l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0l2) = (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\wedge(\forall V1h \in ty\_2Erealax\_2Ereal.(\forall V2t \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V3l2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal))\ V2t)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)))\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1h)\ V3l2))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ c\_2Epoly\_2E\_23\_23\ V1h)\ V3l2))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ V2t)\ V3l2)))))))) \tag{34}$$

Assume the following.

$$(\forall V0p2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V1p1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V2h1 \in ty\_2Erealax\_2Ereal.(\forall V3t1 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(\forall V4h2 \in ty\_2Erealax\_2Ereal.(\forall V5t2 \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).(((ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))\ V0p2) = V0p2)\wedge(((ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V1p1)\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal)) = V1p1)\wedge((ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V2h1)\ V3t1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ V4h2)\ V5t2)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V2h1)\ V4h2))\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_add\ V3t1)\ V5t2)))))))))) \tag{35}$$

Theorem 1

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1q \in (ty\_2Elist\_2Elist\ ty\_2Erealax\_2Ereal).((ap\ (c\_2Elist\_2ELENGTH\ ty\_2Erealax\_2Ereal)\ (ap\ (ap\ c\_2Epoly\_2Epoly\_\_mul\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Erealax\_2Ereal)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\ ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (c\_2Elist\_2ENIL\ ty\_2Erealax\_2Ereal))))\ V1q)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (c\_2Elist\_2ELENGTH\ ty\_2Erealax\_2Ereal)\ V1q)))))$$