

thm_2Epoly_2EPOLY__LINEAR__REM
(TMGntk9kxbDDdFnk5WbWrMe5Up1Yx2N5wc1)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ETL A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (2)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EHD A_27a \in (A_27a)^{(ty_2Elist_2Elist A_27a)} \quad (3)$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P) P)))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_3D (2^{A_27a}) P) P))))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Epoly_2Epoly_add : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_add \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)} \quad (5)$$

Let $c_2Epoly_2E_23_23 : \iota$ be given. Assume the following.

$$c_2Epoly_2E_23_23 \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)} \quad (6)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)} \quad (7)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)} \quad (8)$$

Let $c_2Epoly_2Epoly_mul : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_mul \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)})^{(ty_2Erealax_2Ereal)} \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 13 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (16)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (17)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (18)$$

Definition 15 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ($

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 16 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (21)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (22)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (23)$$

Definition 17 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 20 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (24)$$

Definition 21 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (25)$$

Definition 22 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2EHD A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = V0h))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist A_27a).((ap (c_2Elist_2ETL A_27a) (ap (ap (c_2Elist_2ECONS A_27a) V0h) V1t)) = V1t))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}).(((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist A_27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a.(p (ap V0P (ap (ap (c_2Elist_2ECONS A_27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist A_27a).(p (ap V0P V3l)))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a. (\forall V1a1 \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (\forall V2a0_27 \in A_27a. (\forall V3a1_27 \in \\
& \quad (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\
& \quad V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = \\
& \quad V2a0_27) \wedge (V1a1 = V3a1_27))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\
& \quad A_27a). (\forall V1a0 \in A_27a. (\neg((ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\
& \quad V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). ((ap \\
& \quad (ap\ c_2Epoly_2Epoly_add\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal) \\
& \quad V0l2) = V0l2)) \wedge (\forall V1h \in ty_2Erealax_2Ereal. (\forall V2t \in \\
& \quad (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (\forall V3l2 \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal). ((ap\ (ap\ c_2Epoly_2Epoly_add\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad ty_2Erealax_2Ereal)\ V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (ty_2Elist_2Elist\ ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Emin_2E_3D \\
& \quad (ty_2Elist_2Elist\ ty_2Erealax_2Ereal))\ V3l2)\ (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal))))\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
& \quad V1h)\ V2t))\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ (ap\ (ap \\
& \quad c_2Erealax_2Ereal_add\ V1h)\ (ap\ (c_2Elist_2EHD\ ty_2Erealax_2Ereal) \\
& \quad V3l2))))\ (ap\ (ap\ c_2Epoly_2Epoly_add\ V2t)\ (ap\ (c_2Elist_2ETL\ ty_2Erealax_2Ereal) \\
& \quad V3l2))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0c \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Epoly_2E_23_23 \\
& \quad V0c)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) = (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal))) \wedge (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2h \in \\
& \quad ty_2Erealax_2Ereal. (\forall V3t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\
& \quad ((ap\ (ap\ c_2Epoly_2E_23_23\ V1c)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
& \quad V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ (ap\ (\\
& \quad ap\ c_2Erealax_2Ereal_mul\ V1c)\ V2h))\ (ap\ (ap\ c_2Epoly_2E_23_23 \\
& \quad V1c)\ V3t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).((ap \\
& (ap\ c_2Epoly_2Epoly_mul\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) \\
& \quad V0l2) = (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \wedge (\forall V1h \in \\
& ty_2Erealax_2Ereal.(\forall V2t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\
& \quad (\forall V3l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).((ap\ (\\
ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Elist_2Elist \\
ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Emin_2E3D\ (ty_2Elist_2Elist \\
ty_2Erealax_2Ereal))\ V2t)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \\
(ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ c_2Epoly_2Epoly_add \\
(ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ (c_2Elist_2ECONS \\
ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
(ap\ (ap\ c_2Epoly_2Epoly_mul\ V2t)\ V3l2))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1p1 \in \\
& (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V2h1 \in ty_2Erealax_2Ereal. \\
& \quad (\forall V3t1 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V4h2 \in \\
& ty_2Erealax_2Ereal.(\forall V5t2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\
& \quad (((ap\ (ap\ c_2Epoly_2Epoly_add\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) \\
V0p2) = V0p2) \wedge (((ap\ (ap\ c_2Epoly_2Epoly_add\ V1p1)\ (c_2Elist_2ENIL \\
ty_2Erealax_2Ereal)) = V1p1) \wedge ((ap\ (ap\ c_2Epoly_2Epoly_add\ (\\
ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V2h1)\ V3t1))\ (ap\ (\\
ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V4h2)\ V5t2)) = (ap\ (ap \\
(c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ (ap\ (ap\ c_2Erealax_2Ereal_add \\
V2h1)\ V4h2))\ (ap\ (ap\ c_2Epoly_2Epoly_add\ V3t1)\ V5t2))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1h1 \in \\
& ty_2Erealax_2Ereal.(\forall V2k1 \in ty_2Erealax_2Ereal.(\forall V3t1 \in \\
& (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(((ap\ (ap\ c_2Epoly_2Epoly_mul \\
& \quad (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))\ V0p2) = (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal)) \wedge (((ap\ (ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap \\
& \quad (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V1h1)\ (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal)))\ V0p2) = (ap\ (ap\ c_2Epoly_2E_23_23\ V1h1) \\
& \quad V0p2)) \wedge ((ap\ (ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad ty_2Erealax_2Ereal)\ V1h1)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
& \quad V2k1)\ V3t1)))\ V0p2) = (ap\ (ap\ c_2Epoly_2Epoly_add\ (ap\ (ap\ c_2Epoly_2E_23_23 \\
& \quad V1h1)\ V0p2))\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ (ap \\
& \quad c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Epoly_2Epoly_mul \\
& \quad (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V2k1)\ V3t1))\ V0p2))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad V1y) V0x))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg \\
& \quad V0x) (ap c_2Erealax_2Ereal_neg V1y))))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \quad (58)$$

Assume the following.

$$(\forall V0y \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte V0y) V1x)))))) \quad (59)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) (ap (ap c_2Erealax_2Ereal_add V0x) V1y)))))) \quad (60)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))))) \quad (61)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg (ap c_2Erealax_2Ereal_neg V0x)) = V0x)) \quad (62)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{65}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{71}$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1t \in (ty_2Elist_2Elist \\ & ty_2Erealax_2Ereal). (\forall V2h \in ty_2Erealax_2Ereal. (\exists V3q \in \\ & (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\exists V4r \in ty_2Erealax_2Ereal. \\ & ((ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) V2h) V1t) = (ap (\\ & ap c_2Epoly_2Epoly_add (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) \\ & V4r) (c_2Elist_2ENIL ty_2Erealax_2Ereal))) (ap (ap c_2Epoly_2Epoly_mul \\ & (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) (ap c_2Erealax_2Ereal_neg \\ & V0a)) (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) (ap c_2Ereal_2Ereal_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\ & (c_2Elist_2ENIL ty_2Erealax_2Ereal)))))) V3q)))))) \end{aligned}$$