

thm_2Epoly_2EPOLY__MUL
(TMaUksd3CF9nkEnF8SC21rK773oUq7Mxeff)

October 26, 2020

Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A \cdot 27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A \cdot 27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. \lambda a. (\lambda V2t2 \in A. \lambda a. (\text{ap } (\text{c_2Emin_2E_40 } A) (\lambda V3t3 \in 2. V3t3))))$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (1)$$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a. \text{nonempty } A \Rightarrow \text{c_2Elist_2ECONS } A \Rightarrow a \in (((\text{ty_2Elist_2Elist } A) (\text{ty_2Elist_2Elist } A)) (\text{ty_2Elist_2Elist } A)) \quad (2)$$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a. \text{nonempty } A \Rightarrow \text{c_2Elist_2ENIL } A \Rightarrow a \in (\text{ty_2Elist_2Elist } A) \quad (3)$$

Let `ty_2Erealx_2Ereal` : ι be given. Assume the following.

$$\text{nonempty } (\text{ty_2Erealx_2Ereal } A) \quad (4)$$

Let $c_2Epoly_2Epoly_mul : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_mul \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)} \quad (5)$$

Let $c_2Epoly_2Epoly_add : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_add \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)} \quad (6)$$

Let $c_2Epoly_2E_23_23 : \iota$ be given. Assume the following.

$$c_2Epoly_2E_23_23 \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)} \quad (7)$$

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)} \quad (8)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ n))\ c_2Enum_2E0$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (15)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (16)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (17)$$

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ t))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 15 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (19)$$

Let $c_2Erealax_2Etreallneg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallneg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (20)$$

Let $c_2Erealax_2Etrealleq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealleq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (21)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (22)$$

Definition 16 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 17 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Definition 18 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (23)$$

Definition 20 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (24)$$

Definition 21 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(p\ V1t2 \Rightarrow p\ V2t2)))) \Rightarrow p\ V0t1))$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a). (p\ (ap\ V0P\ V3l)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1a0 \in A_27a. (\neg((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a)))))) \quad (38)$$

Assume the following.

$$((\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c_2Epoly_2Epoly\ (c_2Elist_2ENIL\ ty_2Erealx_2Ereal))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \wedge (\forall V1h \in ty_2Erealx_2Ereal. (\forall V2t \in (ty_2Elist_2Elist\ ty_2Erealx_2Ereal). (\forall V3x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealx_2Ereal)\ V1h)\ V2t))\ V3x) = (ap\ (ap\ c_2Erealx_2Ereal_add\ V1h)\ (ap\ (ap\ c_2Erealx_2Ereal_mul\ V3x)\ (ap\ (ap\ c_2Epoly_2Epoly\ V2t)\ V3x))))))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).((ap \\
& (ap\ c_2Epoly_2Epoly_mul\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) \\
& V0l2) = (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \wedge (\forall V1h \in \\
& ty_2Erealax_2Ereal.(\forall V2t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\
& (\forall V3l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).((ap\ (\\
ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\
V1h) V2t)) V3l2) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Elist_2Elist \\
ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Elist_2Elist \\
ty_2Erealax_2Ereal))\ V2t)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \\
(ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ c_2Epoly_2Epoly_add \\
(ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ (c_2Elist_2ECONS \\
ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\
(ap\ (ap\ c_2Epoly_2Epoly_mul\ V2t)\ V3l2)))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p1 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1p2 \in \\
& (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V2x \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ c_2Epoly_2Epoly_add\ V0p1)\ V1p2)) \\
V2x) = (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap\ c_2Epoly_2Epoly\ V0p1) \\
V2x))\ (ap\ (ap\ c_2Epoly_2Epoly\ V1p2)\ V2x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1c \in \\
& ty_2Erealax_2Ereal.(\forall V2x \in ty_2Erealax_2Ereal.((ap\ (\\
ap\ c_2Epoly_2Epoly\ (ap\ (ap\ c_2Epoly_2E_23_23\ V1c)\ V0p)) V2x) = (\\
ap\ (ap\ c_2Erealax_2Ereal_mul\ V1c)\ (ap\ (ap\ c_2Epoly_2Epoly\ V0p) \\
V2x))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_add \\
V1y)\ V0x))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_add \\
V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_add \\
(ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))\ V2z))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_add \\
(ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)\ V0x) = V0x))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul V1y) V0x)))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z)))))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x)) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y)))))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x) V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \quad (57)$$

Assume the following.

$$(\forall V0y \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte V0y) V1x)))))) \quad (58)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) (ap (ap c_2Erealax_2Ereal_add V0x) V1y)))))) \quad (59)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x) (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))))) \quad (60)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg (ap c_2Erealax_2Ereal_neg V0x) = V0x)) \quad (61)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{65}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (71)$$

Theorem 1

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1p1 \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V2p2 \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). ((ap (ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_mul V1p1) V2p2)) V0x) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Epoly_2Epoly V1p1) V0x)) (ap (ap c_2Epoly_2Epoly V2p2) V0x)))))))$$