

thm_2Epoly_2EPOLY__MUL__CLAUSES (TMGid9UeJupCkx84gKzJzuncFkoVWxAGwpp)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $c_2Enum_2E_2ZERO_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ZERO_2REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2E_2ABS_2num : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ABS_2num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define $c_2Enum_2E_20$ to be $(ap c_2Enum_2E_2ABS_2num c_2Enum_2E_2ZERO_2REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_2num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_2num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Epoly_2Epoly_add : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_add \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{ty_2Elist_2Elist\ ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal} \quad (7)$$

Let $c_2Epoly_2E_23_23 : \iota$ be given. Assume the following.

$$c_2Epoly_2E_23_23 \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{ty_2Elist_2Elist\ ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal} \quad (8)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{ty_2Elist_2Elist\ A_27a})^{A_27a})^{A_27a} \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)^{A_27a} \quad (10)$$

Let $c_2Epoly_2Epoly_mul : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_mul \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{ty_2Elist_2Elist\ ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1a0 \in A_27a. (\neg((ap\ (ap\ (c_2Elist_2ECONS\ A_27a) \\ V1a0)\ V0a1) = (c_2Elist_2ENIL\ A_27a)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} ((\forall V0l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). ((ap \\ (ap\ c_2Epoly_2Epoly_mul\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal)) \\ V0l2) = (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \wedge (\forall V1h \in \\ ty_2Erealax_2Ereal. (\forall V2t \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). \\ (\forall V3l2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). ((ap\ (\\ ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\ V1h)\ V2t))\ V3l2) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Elist_2Elist \\ ty_2Erealax_2Ereal))\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Elist_2Elist \\ ty_2Erealax_2Ereal))\ V2t)\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))) \\ (ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ c_2Epoly_2Epoly_add \\ (ap\ (ap\ c_2Epoly_2E_23_23\ V1h)\ V3l2))\ (ap\ (ap\ (c_2Elist_2ECONS \\ ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \\ (ap\ (ap\ c_2Epoly_2Epoly_mul\ V2t)\ V3l2)))))))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} (\forall V0p2 \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (\forall V1h1 \in \\ ty_2Erealax_2Ereal. (\forall V2k1 \in ty_2Erealax_2Ereal. (\forall V3t1 \in \\ (ty_2Elist_2Elist\ ty_2Erealax_2Ereal). (((ap\ (ap\ c_2Epoly_2Epoly_mul \\ (c_2Elist_2ENIL\ ty_2Erealax_2Ereal))\ V0p2) = (c_2Elist_2ENIL \\ ty_2Erealax_2Ereal)) \wedge (((ap\ (ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap \\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V1h1)\ (c_2Elist_2ENIL \\ ty_2Erealax_2Ereal)))\ V0p2) = (ap\ (ap\ c_2Epoly_2E_23_23\ V1h1) \\ V0p2)) \wedge ((ap\ (ap\ c_2Epoly_2Epoly_mul\ (ap\ (ap\ (c_2Elist_2ECONS \\ ty_2Erealax_2Ereal)\ V1h1)\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal) \\ V2k1)\ V3t1)))\ V0p2) = (ap\ (ap\ c_2Epoly_2Epoly_add\ (ap\ (ap\ c_2Epoly_2E_23_23 \\ V1h1)\ V0p2))\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ (ap \\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ (ap\ (ap\ c_2Epoly_2Epoly_mul \\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealax_2Ereal)\ V2k1)\ V3t1))\ V0p2)))))))))) \end{aligned}$$