

thm_2Epoly_2EPOLY__MVT
(TMdhp6kCanxeYLuBdnUZ8zduCQvunmvfBAy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (5)$$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (7)$$

Definition 9 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 10 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (8)$$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (9)$$

Definition 12 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (10)$$

Definition 15 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Elist_2ETL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ETL\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Epoly_2Epoly_diff_aux : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_diff_aux \in (((ty_2Elist_2Elist\ ty_2Erealax_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealax_2Ereal)})^{ty_2Elist_2Elist\ ty_2Erealax_2Ereal}) \quad (19)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (20)$$

Definition 23 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 24 We define c_Epoly_Ediff to be $\lambda V0l \in (ty_Elist_Elist\ ty_Erealax_Ereal). (ap (ap (ap$

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (21)$$

Let $c_Erealax_Etrealm_inv : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (22)$$

Definition 25 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal. (ap\ c_Erealax_Ereal_ABS$

Definition 26 We define $c_Ereal_E_EF$ to be $\lambda V0x \in ty_Erealax_Ereal. \lambda V1y \in ty_Erealax_Ereal. ($

Definition 27 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal. (ap (ap (ap (c_Ebool_ECOND$

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_Epair_EESND\ A_27a\ A_27b \in (A_27b)^{(ty_Epair_Eprod\ A_27a\ A_27b)} \quad (23)$$

Let $c_Epair_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_Epair_EFAST\ A_27a\ A_27b \in (A_27a)^{(ty_Epair_Eprod\ A_27a\ A_27b)} \quad (24)$$

Definition 28 We define $c_Epair_EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a}$

Let $ty_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_Emetric_Emetric\ A0) \quad (25)$$

Let $c_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Emetric_Emetric\ A_27a \in ((ty_Emetric_Emetric\ A_27a)^{(ty_Erealax_Ereal)^{(ty_Epair_Eprod\ A_27a\ A_27a)}}) \quad (26)$$

Definition 29 We define $c_Emetric_Emr1$ to be $(ap (c_Emetric_Emetric\ ty_Erealax_Ereal) (ap (c$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (27)$$

Definition 30 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Enets_2Etendsto A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod (ty_2Emetric_2Emetric A_27a))}) \quad (28)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Emetric_2Emetric A_27a)}) \quad (29)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (30)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (31)$$

Definition 31 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (32)$$

Definition 32 We define $c_2Elim_2Etends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$.

Definition 33 We define c_2Elim_2Ediff to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1l \in ty_2Erealax_2Ereal$.

Definition 34 We define $c_2Elim_2Edifferentiable$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1l \in ty_2Erealax_2Ereal$.

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{(ty_2Elist_2Elist ty_2Erealax_2Ereal)}) \quad (33)$$

Definition 35 We define $c_2Elim_2Econtl$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1x \in ty_2Erealax_2Ereal$.

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& p V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\
& ty_2Erealax_2Ereal.(\forall V2m \in ty_2Erealax_2Ereal.(\forall V3x \in \\
& ty_2Erealax_2Ereal.(((p (ap (ap (ap (ap c_2Elim_2Ediff1 V0f) V1l) \\
& V3x)) \wedge (p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2m) V3x))) \Rightarrow (V1l = V2m))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(((p (\\
& ap (ap c_2Erealax_2Ereal_lt V1a) V2b)) \wedge ((\forall V3x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte V1a) V3x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V3x) V2b))) \Rightarrow (p (ap (ap c_2Elim_2Econt1 V0f) V3x)))) \wedge (\forall V4x \in \\
& ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V1a) V4x)) \wedge \\
& (p (ap (ap c_2Erealax_2Ereal_lt V4x) V2b))) \Rightarrow (p (ap (ap c_2Elim_2Edifferentiable \\
& V0f) V4x)))))) \Rightarrow (\exists V5l \in ty_2Erealax_2Ereal.(\exists V6z \in \\
& ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt V1a) V6z)) \wedge \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V6z) V2b)) \wedge ((p (ap (ap (ap c_2Elim_2Ediff1 \\
& V0f) V5l) V6z)) \wedge ((ap (ap c_2Ereal_2Ereal_sub (ap V0f V2b)) (ap \\
& V0f V1a)) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Ereal_2Ereal_sub \\
& V2b) V1a)) V5l))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist ty_2Erealax_2Ereal).(\forall V1x \in \\
& ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Epoly_2Epoly V0l) V2x))) (ap (ap c_2Epoly_2Epoly (ap \\
& c_2Epoly_2Ediff V0l)) V1x)) V1x)))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist ty_2Erealax_2Ereal).(\forall V1x \in \\
& ty_2Erealax_2Ereal.(p (ap (ap c_2Elim_2Edifferentiable (\lambda V2x \in \\
& ty_2Erealax_2Ereal.(ap (ap c_2Epoly_2Epoly V0l) V2x))) V1x)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1x \in \\
& ty_2Erealax_2Ereal.(p (ap (ap\ c_2Elim_2Econtl\ (\lambda V2x \in ty_2Erealax_2Ereal. \\
& (ap (ap\ c_2Epoly_2Epoly\ V0l)\ V2x)))\ V1x)))) \\
& \hspace{15em} (43)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist\ ty_2Erealax_2Ereal).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p (ap \\
& (ap\ c_2Erealax_2Ereal_lt\ V1a)\ V2b)) \Rightarrow (\exists V3x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap\ c_2Erealax_2Ereal_lt\ V1a)\ V3x)) \wedge ((p (ap (ap\ c_2Erealax_2Ereal_lt \\
& V3x)\ V2b)) \wedge ((ap (ap\ c_2Ereal_2Ereal_sub\ (ap (ap\ c_2Epoly_2Epoly \\
& V0p)\ V2b)) (ap (ap\ c_2Epoly_2Epoly\ V0p)\ V1a)) = (ap (ap\ c_2Erealax_2Ereal_mul \\
& (ap (ap\ c_2Ereal_2Ereal_sub\ V2b)\ V1a)) (ap (ap\ c_2Epoly_2Epoly \\
& (ap\ c_2Epoly_2Ediff\ V0p))\ V3x))))))))))
\end{aligned}$$