

thm_2Epoly_2EPOLY__ORDER__EXISTS (TMVY5KdDRjGZKS1pY79cRLcByroeveP2yta)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 3 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

Definition 20 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 21 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ c_2Enum_2ESUC\ (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 22 We define `c_2Earithmetic_2EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2) V0n)$

Definition 23 We define `c_2Enumeral_2EiDUB` to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2) V0x)$

Definition 24 We define `c_2Enumeral_2EiZ` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (11)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (12)$$

Let $c_2Epoly_2Epoly_exp : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_exp \in (((ty_2Elist_2Elist\ ty_2Erealx_2Ereal)^{ty_2Enum_2Enum})^{(ty_2Elist_2Elist\ ty_2Erealx_2Ereal)}) \quad (13)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (14)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (15)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (16)$$

Let $c_2Epoly_2Epoly_mul : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly_mul \in (((ty_2Elist_2Elist\ ty_2Erealx_2Ereal)^{(ty_2Elist_2Elist\ ty_2Erealx_2Ereal)})^{(ty_2Elist_2Elist\ ty_2Erealx_2Ereal)}) \quad (17)$$

Let $c_2Epoly_2Epoly : \iota$ be given. Assume the following.

$$c_2Epoly_2Epoly \in ((ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal})^{(ty_2Elist_2Elist\ ty_2Erealx_2Ereal)}) \quad (18)$$

Definition 25 We define `c_2Epoly_2Epoly_divides` to be $\lambda V0p1 \in (ty_2Elist_2Elist\ ty_2Erealx_2Ereal).$

Definition 26 We define `c_2Earithmetic_2EZERO` to be `c_2Enum_2E0`.

Definition 27 We define `c_2Earithmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2) V0n)$

Definition 28 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (19)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (20)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (21)$$

Definition 29 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ a)))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (22)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (c_2Etreall_lt\ T1\ T2)$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (23)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (24)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (25)$$

Definition 31 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal). (c_2Erealax_2Ereal_ABS_CLASS\ r)$

Definition 32 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal_neg\ T1)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 33 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal. (c_2Etreall_add\ T1\ T2)$

Definition 34 We define $c_2Earithmetric_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Earithmetric_2E_3C_3D\ m\ n)$

Definition 35 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal. \lambda V1y \in ty_Erealax_Ereal.$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (27)$$

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Enum_Enum}) \quad (28)$$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})(ty_Epair_Eprod\ ty_Ehreal_Ehreal)) \quad (29)$$

Definition 36 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal. \lambda V1T2 \in ty_Erealax_Ereal.$

Assume the following.

$$(\forall V0n \in ty_Enum_Enum. (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ c_Enum_E0)\ V0n))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty_Enum_Enum. (\forall V1n \in ty_Enum_Enum. ((ap\ (ap\ c_Earithmetic_E_2B\ V0m)\ V1n) = c_Enum_E0) \Leftrightarrow ((V0m = c_Enum_E0) \wedge (V1n = c_Enum_E0)))) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty_Enum_Enum. (\forall V1n \in ty_Enum_Enum. ((V0m = V1n) \Leftrightarrow ((p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V1n)\ V0m)))))) \quad (32)$$

Assume the following.

$$((\forall V0n \in ty_Enum_Enum. ((p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V0n)\ c_Enum_E0)) \Leftrightarrow (V0n = c_Enum_E0))) \wedge (\forall V1m \in ty_Enum_Enum. (\forall V2n \in ty_Enum_Enum. ((p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V1m)\ (ap\ c_Enum_ESUC\ V2n))) \Leftrightarrow ((V1m = (ap\ c_Enum_ESUC\ V2n)) \vee (p\ (ap\ (ap\ c_Earithmetic_E_3C_3D\ V1m)\ V2n))))))) \quad (33)$$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((p \ V0t) \vee (\neg(p \ V0t)))) \quad (37)$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. (((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (43)$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (44)$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty \ A_27a \Rightarrow \forall A_27b. nonempty \ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((\forall V2x \in \\ & A_27a. ((ap \ V0f \ V2x) = (ap \ V1g \ V2x))) \Rightarrow (V0f = V1g)))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((p \ V0P) \vee (\exists V2x \in A_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in A_27a.((\forall V2x \in A_27a.((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ (ap \ V0f \ V1v)))) \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\exists V2y \in \\ A_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}).(\\ \forall V4x \in A_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2El \\ A_27a)\ V0x) = V0x)) \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).(((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l) = c_2Enum_2E0) \Leftrightarrow (\\ V0l = (c_2Elist_2ENIL\ A_27a)))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ (\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ V0P\ V2n)))) \end{aligned} \quad (60)$$

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& \quad (ap c_2Earithmic_2ENUMERAL V2n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V3m))) = (ap c_2Earithmic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A (\\
& \quad ap c_2Earithmic_2ENUMERAL V6n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V7m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& \quad (ap c_2Earithmic_2ENUMERAL V10n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V11m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmic_2EEXP c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Earithmic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2EEXP (ap c_2Earithmic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmic_2ENUMERAL V16m))) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad c_2Earithmic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmic_2ENUMERAL V17n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmic_2ENUMERAL V18n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V32n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D c_2Earithmic_2EZERO) \\
& \quad V32n)))))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)))))) \wedge ((\forall V34n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)))))) \wedge ((\forall V35n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V35n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V35n)) c_2Enum_2E0)))))) \wedge ((\forall V36n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)))))) \wedge ((\forall V37n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)))))) \wedge ((\forall V38n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V38n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V38n)) c_2Enum_2E0)))))) \wedge ((\forall V39n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)))))) \wedge ((\forall V40n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)))))) \wedge ((\forall V41n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V41n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V41n)) c_2Enum_2E0)))))) \wedge ((\forall V42n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)))))) \wedge ((\forall V43n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V43n)) c_2Enum_2E0)) \Leftrightarrow (p$$

[illegible]

(62)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{63}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n)) = (ap\ c_2Earithmic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmic_2EBIT2\ V0n)) = (ap \\
& c_2Earithmic_2EBIT2\ (ap\ c_2Earithmic_2EBIT1\ V0n))) \wedge ((ap \\
& c_2Enumeral_2EiDUB\ c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO)))) \\
& \tag{64}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ (ap\ c_2Earithmic_2E_2A\ c_2Earithmic_2EZERO)\ V0n) = c_2Earithmic_2EZERO) \wedge \\
& (((ap\ (ap\ c_2Earithmic_2E_2A\ V0n)\ c_2Earithmic_2EZERO) = \\
& c_2Earithmic_2EZERO) \wedge (((ap\ (ap\ c_2Earithmic_2E_2A\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ V1m) = (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmic_2E_2B \\
& (ap\ c_2Enumeral_2EiDUB\ (ap\ (ap\ c_2Earithmic_2E_2A\ V0n)\ V1m))) \\
& V1m))) \wedge ((ap\ (ap\ c_2Earithmic_2E_2A\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ V1m) = (ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Enumeral_2EiZ\ (ap\ (ap \\
& c_2Earithmic_2E_2B\ (ap\ (ap\ c_2Earithmic_2E_2A\ V0n)\ V1m)) \\
& V1m))))))))) \\
& \tag{65}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Erealx_2Ereal. ((ap\ (ap\ c_2Epoly_2Epoly\ (\\
& c_2Elist_2ENIL\ ty_2Erealx_2Ereal))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \wedge (\forall V1h \in ty_2Erealx_2Ereal. (\forall V2t \in \\
& (ty_2Elist_2Elist\ ty_2Erealx_2Ereal). (\forall V3x \in ty_2Erealx_2Ereal. \\
& ((ap\ (ap\ c_2Epoly_2Epoly\ (ap\ (ap\ (c_2Elist_2ECONS\ ty_2Erealx_2Ereal \\
& V1h)\ V2t))\ V3x) = (ap\ (ap\ c_2Erealx_2Ereal_add\ V1h)\ (ap\ (ap\ c_2Erealx_2Ereal_mul \\
& V3x)\ (ap\ (ap\ c_2Epoly_2Epoly\ V2t)\ V3x))))))))) \\
& \tag{66}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0p \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal).((ap \ (\\
& \quad ap \ c_2Epoly_2Epoly_exp \ V0p) \ c_2Enum_2E0) = (ap \ (ap \ (c_2Elist_2ECONS \\
& \quad ty_2Erealax_2Ereal) \ (ap \ c_2Ereal_2Ereal_of_num \ (ap \ c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap \ c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))) \ (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal)))) \wedge (\forall V1p \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal). \\
& \quad (\forall V2n \in ty_2Enum_2Enum.((ap \ (ap \ c_2Epoly_2Epoly_exp \ V1p) \\
& \quad \quad (ap \ c_2Enum_2ESUC \ V2n)) = (ap \ (ap \ c_2Epoly_2Epoly_mul \ V1p) \ (ap \\
& \quad \quad \quad (ap \ c_2Epoly_2Epoly_exp \ V1p) \ V2n))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1p1 \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal).(\forall V2p2 \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal). \\
& \quad ((ap \ (ap \ c_2Epoly_2Epoly \ (ap \ (ap \ c_2Epoly_2Epoly_mul \ V1p1) \ V2p2)) \\
& \quad \quad V0x) = (ap \ (ap \ c_2Erealax_2Ereal_mul \ (ap \ (ap \ c_2Epoly_2Epoly \ V1p1) \\
& \quad \quad \quad V0x)) \ (ap \ (ap \ c_2Epoly_2Epoly \ V2p2) \ V0x))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal).(\forall V1q \in \\
& \quad (ty_2Elist_2Elist \ ty_2Erealax_2Ereal).(\forall V2r \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal).((ap \ c_2Epoly_2Epoly \ (ap \ (ap \ c_2Epoly_2Epoly_mul \\
& \quad \quad V0p) \ (ap \ (ap \ c_2Epoly_2Epoly_mul \ V1q) \ V2r))) = (ap \ c_2Epoly_2Epoly \\
& \quad \quad (ap \ (ap \ c_2Epoly_2Epoly_mul \ (ap \ (ap \ c_2Epoly_2Epoly_mul \ V0p) \\
& \quad \quad \quad V1q)) \ V2r))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal.(\forall V1p \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal).((ap \ (ap \ c_2Epoly_2Epoly \ V1p) \ V0a) = (ap \\
& \quad c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \Leftrightarrow ((V1p = (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal)) \vee (\exists V2q \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal). \\
& \quad (V1p = (ap \ (ap \ c_2Epoly_2Epoly_mul \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& \quad \quad (ap \ c_2Erealax_2Ereal_neg \ V0a)) \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& \quad \quad \quad (ap \ c_2Ereal_2Ereal_of_num \ (ap \ c_2Earithmetic_2ENUMERAL \ (\\
& \quad \quad \quad ap \ c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))) \ (c_2Elist_2ENIL \\
& \quad \quad \quad ty_2Erealax_2Ereal)))) \ V2q))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1q \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal). ((ap (c_2Elist_2ELENGTH ty_2Erealax_2Ereal) \\
& \quad (ap (ap c_2Epoly_2Epoly_mul (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) \\
& \quad (ap c_2Erealax_2Ereal_neg V0a)) (ap (ap (c_2Elist_2ECONS ty_2Erealax_2Ereal) \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal)))) V1q)) = (ap c_2Enum_2ESUC (ap (c_2Elist_2ELENGTH \\
& \quad ty_2Erealax_2Ereal) V1q))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V1q \in \\
& \quad (ty_2Elist_2Elist ty_2Erealax_2Ereal). (((ap c_2Epoly_2Epoly \\
& \quad (ap (ap c_2Epoly_2Epoly_mul V0p) V1q)) = (ap c_2Epoly_2Epoly (\\
& \quad \quad c_2Elist_2ENIL ty_2Erealax_2Ereal)))) \Leftrightarrow (((ap c_2Epoly_2Epoly \\
& \quad V0p) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL ty_2Erealax_2Ereal))) \vee \\
& \quad ((ap c_2Epoly_2Epoly V1q) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL \\
& \quad \quad ty_2Erealax_2Ereal))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V1q \in \\
& \quad (ty_2Elist_2Elist ty_2Erealax_2Ereal). (\forall V2r \in (ty_2Elist_2Elist \\
& \quad ty_2Erealax_2Ereal). (((ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_mul \\
& \quad \quad V0p) V1q)) = (ap c_2Epoly_2Epoly (ap (ap c_2Epoly_2Epoly_mul V0p) \\
& \quad \quad V2r))) \Leftrightarrow (((ap c_2Epoly_2Epoly V0p) = (ap c_2Epoly_2Epoly (c_2Elist_2ENIL \\
& \quad ty_2Erealax_2Ereal))) \vee ((ap c_2Epoly_2Epoly V1q) = (ap c_2Epoly_2Epoly \\
& \quad \quad V2r))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap c_2Enum_2ESUC V0m) = (ap c_2Enum_2ESUC V1n)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A. 27a.nonempty A. 27a \Rightarrow (\forall V0e \in A. 27a. (\forall V1f \in \\
& \quad ((A. 27a^{A. 27a})^{ty_2Enum_2Enum}). (\exists V2fn \in (A. 27a^{ty_2Enum_2Enum}). \\
& \quad (((ap V2fn c_2Enum_2E0) = V0e) \wedge (\forall V3n \in ty_2Enum_2Enum. (\\
& \quad (ap V2fn (ap c_2Enum_2ESUC V3n)) = (ap (ap V1f V3n) (ap V2fn V3n))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad \quad V1y) V0x))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V0x))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{84}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x)) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y)))))) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (90)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Ereal_neg V1y)) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))) \quad (92)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lte V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x)))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y)))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x)))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z)))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))
\end{aligned} \tag{102}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{103}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B)))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B)))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{106}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \tag{109}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \tag{110}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \\
& \tag{111}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \\
& \tag{112}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in ty_2Erealax_2Ereal. (\forall V1d \in ty_2Enum_2Enum. \\
& (\forall V2p \in (ty_2Elist_2Elist \ ty_2Erealax_2Ereal). (((ap \\
& (c_2Elist_2ELENGTH \ ty_2Erealax_2Ereal) \ V2p) = V1d) \wedge (\neg((ap \ c_2Epoly_2Epoly \\
& V2p) = (ap \ c_2Epoly_2Epoly \ (c_2Elist_2ENIL \ ty_2Erealax_2Ereal)))))) \Rightarrow \\
& (\exists V3n \in ty_2Enum_2Enum. ((p \ (ap \ (ap \ c_2Epoly_2Epoly_divides \\
& (ap \ (ap \ c_2Epoly_2Epoly_exp \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Erealax_2Ereal_neg \ V0a)) \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Ereal_2Ereal_of_num \ (ap \ c_2Earithmetic_2ENUMERAL \ (\\
& ap \ c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))) \ (c_2Elist_2ENIL \\
& ty_2Erealax_2Ereal)))) \ V3n)) \ V2p)) \wedge (\neg(p \ (ap \ (ap \ c_2Epoly_2Epoly_divides \\
& (ap \ (ap \ c_2Epoly_2Epoly_exp \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Erealax_2Ereal_neg \ V0a)) \ (ap \ (ap \ (c_2Elist_2ECONS \ ty_2Erealax_2Ereal) \\
& (ap \ c_2Ereal_2Ereal_of_num \ (ap \ c_2Earithmetic_2ENUMERAL \ (\\
& ap \ c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))) \ (c_2Elist_2ENIL \\
& ty_2Erealax_2Ereal)))) \ (ap \ c_2Enum_2ESUC \ V3n)) \ V2p)))))) \\
& \tag{Theorem 1}
\end{aligned}$$