

thm_2Eposet_2Ecomplete_top (TM- MMD3JiKutN25fGLFz7GgpqyHXKcWukski)

October 26, 2020

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_1.nonempty\ A_1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A_0\ A_1) \quad (1)$$

Let $c_2Eposet_2Eposet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Eposet\ A_27a \in (2^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (V0P))))$

Definition 4 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define c_2Ebool_2E27E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F))$

Let $c_2Eposet_2E2top : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2E2top\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (3)$$

Definition 7 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2E2ABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2E2ABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$
 Let $c_2Eposet_2Eglb : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eposet_2Eglb A_27a \in (((2^{A_27a})^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) ((2^{A_27a})^A))} \quad (5)$$

Let $c_2Eposet_2Elub : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eposet_2Elub A_27a \in (((2^{A_27a})^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) ((2^{A_27a})^A))} \quad (6)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then (the } (\lambda x. x \in A \wedge p$
 of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eposet_2Ecomplete$ to be $\lambda A_27a : \iota. \lambda V0p \in (ty_2Epair_2Eprod (2^{A_27a}) ((2^A$
 Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (9) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (10) \end{aligned}$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (13) \end{aligned}$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod \ A_27a \ A_27b).(\exists V1q \in A_27a. \\ (\exists V2r \in A_27b.(V0x = (ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \\ V1q) \ V2r)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1r \in \\ ((2^{A_27a})^{A_27a}).(\forall V2x \in A_27a.((p \ (ap \ (ap \ (c_2Eposet_2Etop \\ A_27a) \ (ap \ (ap \ (c_2Epair_2E_2C \ (2^{A_27a}) \ ((2^{A_27a})^{A_27a})) \ V0s) \\ V1r)) \ V2x)) \Leftrightarrow ((p \ (ap \ V0s \ V2x)) \wedge (\forall V3y \in A_27a.((p \ (ap \ V0s \ V3y)) \Rightarrow \\ (p \ (ap \ (ap \ V1r \ V3y) \ V2x)))))))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1r \in \\ ((2^{A_27a})^{A_27a}).(\forall V2p \in (2^{A_27a}).(\forall V3x \in A_27a. \\ ((p \ (ap \ (ap \ (ap \ (c_2Eposet_2Elub \ A_27a) \ (ap \ (ap \ (c_2Epair_2E_2C \\ (2^{A_27a}) \ ((2^{A_27a})^{A_27a})) \ V0s) \ V1r)) \ V2p) \ V3x)) \Leftrightarrow ((p \ (ap \ V0s \ V3x)) \wedge \\ ((\forall V4y \in A_27a.(((p \ (ap \ V0s \ V4y)) \wedge (p \ (ap \ V2p \ V4y))) \Rightarrow (p \ (ap \\ (ap \ V1r \ V4y) \ V3x)))) \wedge (\forall V5z \in A_27a.(((p \ (ap \ V0s \ V5z)) \wedge (\forall V6y \in \\ A_27a.(((p \ (ap \ V0s \ V6y)) \wedge (p \ (ap \ V2p \ V6y))) \Rightarrow (p \ (ap \ (ap \ V1r \ V6y) \ V5z)))))) \Rightarrow \\ (p \ (ap \ (ap \ V1r \ V3x) \ V5z)))))))))) \quad (17)$$

Theorem 1

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ (2^{A_27a}) \ ((2^{A_27a})^{A_27a})).(((p \ (ap \ (c_2Eposet_2Eposet \ A_27a) \\ V0p)) \wedge (p \ (ap \ (c_2Eposet_2Ecomplete \ A_27a) \ V0p))) \Rightarrow (\exists V1x \in \\ A_27a.(p \ (ap \ (ap \ (c_2Eposet_2Etop \ A_27a) \ V0p) \ V1x))))))$$