

thm_2Eposet_2Egfp_unique (TMbxJtR- PqV6X92M4vVkHXFmLLPXsWGvMNMi)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 0t \in 2. V 0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2E` to be $(\lambda V 0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V 0t) (\text{c_2Ebool_2E_2F}))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{1}$$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } A 27a))))$

Let `c_2Eposet_2Eposet` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Eposet_2Eposet } A. 27a \in (2^{(\text{ty_2Epair_2Eprod } (2^{A-27a}) ((2^{A-27a})^{A-27a}))) \tag{2}$$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V 0t 1 \in 2. (\lambda V 1t 2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V 2t \in 2. \text{c_2Ebool_2E_2E } (V 0t V 1t V 2t))))$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EABS_prod } A. 27a A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a A. 27b)^{(2^{A-27b})^{A-27a}}) \tag{3}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Eposet_2Egfp : \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eposet_2Egfp A_27a \in (((2^{A_27a})^{(A_27a)^{A_27a}})^{(ty_2Epair_2Eprod (2^{A_27a}) (2^{A_27a})})) \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod A_27a A_27b).(\exists V1q \in A_27a. \\ & (\exists V2r \in A_27b.(V0x = (ap (ap (c_2Epair_2E_2C A_27a A_27b) \\ & V1q) V2r)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in \\
& ((2^{A_27a})^{A_27a}). ((p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r))) \Leftrightarrow ((\exists V2x \in A_27a. \\
& (p\ (ap\ V0s\ V2x))) \wedge ((\forall V3x \in A_27a. ((p\ (ap\ V0s\ V3x)) \Rightarrow (p\ (ap\ (\\
& ap\ V1r\ V3x)\ V3x)))))) \wedge ((\forall V4x \in A_27a. (\forall V5y \in A_27a. (\\
& ((p\ (ap\ V0s\ V4x)) \wedge ((p\ (ap\ V0s\ V5y)) \wedge ((p\ (ap\ (ap\ V1r\ V4x)\ V5y)) \wedge (p\ (\\
& ap\ (ap\ V1r\ V5y)\ V4x)))))) \Rightarrow (V4x = V5y)))) \wedge (\forall V6x \in A_27a. (\forall V7y \in \\
& A_27a. (\forall V8z \in A_27a. (((p\ (ap\ V0s\ V6x)) \wedge ((p\ (ap\ V0s\ V7y)) \wedge \\
& ((p\ (ap\ V0s\ V8z)) \wedge ((p\ (ap\ (ap\ V1r\ V6x)\ V7y)) \wedge (p\ (ap\ (ap\ V1r\ V7y)\ V8z)))))) \Rightarrow \\
& (p\ (ap\ (ap\ V1r\ V6x)\ V8z))))))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in \\
& ((2^{A_27a})^{A_27a}). (\forall V2f \in (A_27a)^{A_27a}). (\forall V3x \in A_27a. \\
& ((p\ (ap\ (ap\ (ap\ (c_2Eposet_2Egfp\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\
& (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r))\ V2f)\ V3x)) \Leftrightarrow ((p\ (ap\ V0s\ V3x)) \wedge \\
& (((ap\ V2f\ V3x) = V3x) \wedge (\forall V4y \in A_27a. (((p\ (ap\ V0s\ V4y)) \wedge (p\ (\\
& ap\ (ap\ V1r\ V4y)\ (ap\ V2f\ V4y)))) \Rightarrow (p\ (ap\ (ap\ V1r\ V4y)\ V3x))))))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A_27a})\ ((2^{A_27a})^{A_27a})). (\forall V1f \in (A_27a)^{A_27a}). (\forall V2x \in \\
& A_27a. (\forall V3x_27 \in A_27a. (((p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ \\
& V0p)) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Eposet_2Egfp\ A_27a)\ V0p)\ V1f)\ V2x)) \wedge (\\
& p\ (ap\ (ap\ (ap\ (c_2Eposet_2Egfp\ A_27a)\ V0p)\ V1f)\ V3x_27)))) \Rightarrow (V2x = \\
& V3x_27))))))
\end{aligned}$$