thm_2Eposet_2Eknaster__tarski (TMQuRBDNKp3ZZHLrCNruYfnfvaJsNDYMWUa)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Eposet\_2Elfp : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Elfp\ A\_27a \in (((2^{A\_27a})^{(A\_27a^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_2} \tag{2}$$

Let $c\_2Eposet\_2Egfp : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Egfp\ A\_27a \in (((2^{A\_27a})^{(A\_27a^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A-} \tag{3}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $c\_2Eposet\_2Emonotonic : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Emonotonic\ A\_27a \in ((2^{(A\_27a^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))}) \tag{4}$$

Let $c\_2Eposet\_2Ecarrier : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Ecarrier\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))} \tag{5}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Eposet_2Efunction *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in (2^{A\_27a}).\lambda V1b \in (2^{A\_27b}).$

Let $c\_2Eposet\_2Eglb : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Eglb\ A\_27a \in (((2^{A\_27a})^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_}} \tag{6}$$

Let $c\_2Eposet\_2Elub : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Elub\ A\_27a \in (((2^{A\_27a})^{(2^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_}} \tag{7}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Eposet_2Ecomplete *to be* $\lambda A\_27a : \iota.\lambda V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_2}$

Let $c\_2Eposet\_2Eposet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Eposet\ A\_27a \in (2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))}) \tag{8}$$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t)).$

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P)\vee(\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x))))\Leftrightarrow(\exists V3x \in A\_27a.((p\ V0P)\vee(p\ (ap\ V1Q\ V3x))))))) \tag{19}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{20}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee (p\ V0A))))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y))))\Leftrightarrow(\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a})).(\forall V1f \in (A\_27a^{A\_27a}).(((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (\mathsf{c\_2Eposet\_2Ecomplete}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eposet\_2Efunction}\ A\_27a\ A\_27a)\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ V1f)) \wedge (p\ (ap\ (ap\ (c\_2Eposet\_2Emonotonic\ A\_27a)\ V0p)\ V1f))))) \Rightarrow (\exists V2x \in A\_27a.(p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Elfp\ A\_27a)\ V0p)\ V1f)\ V2x)))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a})).(\forall V1f \in (A\_27a^{A\_27a}).(((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (\mathsf{c\_2Eposet\_2Ecomplete}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eposet\_2Efunction}\ A\_27a\ A\_27a)\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ V1f)) \wedge (p\ (ap\ (ap\ (c\_2Eposet\_2Emonotonic\ A\_27a)\ V0p)\ V1f))))) \Rightarrow (\exists V2x \in A\_27a.(p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Egfp\ A\_27a)\ V0p)\ V1f)\ V2x)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{27}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{30}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{31}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{35}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a})).(\forall V1f \in (A\_27a^{A\_27a}).(((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ V0p))\wedge((p\ (ap\ (\mathsf{c\_2Eposet\_2Ecomplete}\ A\_27a)\ V0p))\wedge((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Eposet\_2Efunction}\ A\_27a\ A\_27a)\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ (ap\ (c\_2Eposet\_2Ecarrier\ A\_27a)\ V0p))\ V1f))\wedge(p\ (ap\ (ap\ (c\_2Eposet\_2Emonotonic\ A\_27a)\ V0p)\ V1f)))))\Rightarrow((\exists V2x \in A\_27a.(p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Elfp\ A\_27a)\ V0p)\ V1f)\ V2x)))\wedge(\exists V3x \in A\_27a.(p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Egfp\ A\_27a)\ V0p)\ V1f)\ V3x)))))))$$