

thm_2Eposet_2Eknaster__tarski__gfp
(TMGPtvmFXLsJr9nQFyR1yKTqdiaz3zprHty)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ecombin_2E_EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 5 We define $c_2Ecombin_2E_ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 6 We define $c_2Ecombin_2E_EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (2^{A_27a}))))$

Definition 9 We define $c_2Eposet_2Efunction$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in (2^{A_27a}).\lambda V1b \in (2^{A_27b}).$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Eposet_2Ecarrier : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eposet_2Ecarrier A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod (2^{A_27a}) ((2^{A_27a})^{A_27a}))) \tag{2}$$

Let $c_2Eposet_2Egblb : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eposet_2Egblb A_27a \in (((2^{A_27a})^{(2^{A_27a})})^{(ty_2Epair_2Eprod (2^{A_27a}) ((2^{A_27a})^{A_27a}))) \tag{3}$$

Let $c_2Eposet_2Elub : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Elub\ A_27a \in (((2^{A_27a})^{(2^{A_27a})})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (4)$$

Let $c_2Eposet_2Ebottom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Ebottom\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (5)$$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 12 We define $c_2Eposet_2Ecomplete$ to be $\lambda A_27a : \iota. \lambda V0p \in (ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^A$

Let $c_2Eposet_2Eposet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Eposet\ A_27a \in (2^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (6)$$

Let $c_2Eposet_2Emonotonic : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Emonotonic\ A_27a \in ((2^{(A_27a)^{A_27a}})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Let $c_2Eposet_2Egfp : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Egfp\ A_27a \in (((2^{A_27a})^{(A_27a)^{A_27a}})^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^A$$
 \quad (9)

Definition 14 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A.27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \vee (\exists V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q)) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b)^{A.27a}. (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & \\ \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. & \\ (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) & \\ V1q)\ V2r)))))) & \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in & \\ ((2^{A_27a})^{A_27a}). ((ap\ (c_2Eposet_2Ecarrier\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C & \\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r)) = V0s))) & \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in & \\ ((2^{A_27a})^{A_27a}). (\forall V2p \in (2^{A_27a}). (\forall V3x \in A_27a. & \\ ((p\ (ap\ (ap\ (ap\ (c_2Eposet_2Elub\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C & \\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r))\ V2p)\ V3x)) \Leftrightarrow ((p\ (ap\ V0s\ V3x)) \wedge & \\ ((\forall V4y \in A_27a. (((p\ (ap\ V0s\ V4y)) \wedge (p\ (ap\ V2p\ V4y))) \Rightarrow (p\ (ap & \\ (ap\ V1r\ V4y)\ V3x)))) \wedge (\forall V5z \in A_27a. (((p\ (ap\ V0s\ V5z)) \wedge (\forall V6y \in & \\ A_27a. (((p\ (ap\ V0s\ V6y)) \wedge (p\ (ap\ V2p\ V6y))) \Rightarrow (p\ (ap\ (ap\ V1r\ V6y)\ V5z)))))) \Rightarrow & \\ (p\ (ap\ (ap\ V1r\ V3x)\ V5z)))))))))) & \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in & \\ ((2^{A_27a})^{A_27a}). (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((& \\ (p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a}) & \\ (2^{A_27a})^{A_27a}))\ V0s)\ V1r))) \wedge ((p\ (ap\ V0s\ V2x)) \wedge ((p\ (ap\ V0s\ V3y)) \wedge & \\ ((p\ (ap\ (ap\ V1r\ V2x)\ V3y)) \wedge (p\ (ap\ (ap\ V1r\ V3y)\ V2x)))))) \Rightarrow (V2x = V3y)))))) & \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in & \\ ((2^{A_27a})^{A_27a}). (\forall V2x \in A_27a. (\forall V3y \in A_27a. (\forall V4z \in & \\ A_27a. (((p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C & \\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r))) \wedge ((p\ (ap\ V0s\ V2x)) \wedge ((p & \\ (ap\ V0s\ V3y)) \wedge ((p\ (ap\ V0s\ V4z)) \wedge ((p\ (ap\ (ap\ V1r\ V2x)\ V3y)) \wedge (p & \\ (ap\ (ap\ V1r\ V3y)\ V4z)))))) \Rightarrow (p\ (ap\ (ap\ V1r\ V2x)\ V4z)))))) & \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\ (2^{A_27a}) ((2^{A_27a})^{A_27a})).((p (ap (c_2Eposet_2Eposet\ A_27a) \\ V0p)) \wedge (p (ap (c_2Eposet_2Ecomplete\ A_27a) V0p))) \Rightarrow (\exists V1x \in \\ A_27a.(p (ap (ap (c_2Eposet_2Ebottom\ A_27a) V0p) V1x)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1r \in \\ ((2^{A_27a})^{A_27a}).(\forall V2f \in (A_27a^{A_27a}).((p (ap (ap (c_2Eposet_2Emonotonic \\ A_27a) (ap (ap (c_2Epair_2E_2C (2^{A_27a}) ((2^{A_27a})^{A_27a})) V0s) \\ V1r)) V2f)) \Leftrightarrow (\forall V3x \in A_27a.(\forall V4y \in A_27a.(((p (ap V0s \\ V3x)) \wedge ((p (ap V0s V4y)) \wedge (p (ap (ap V1r V3x) V4y)))) \Rightarrow (p (ap (ap V1r \\ (ap V2f V3x)) (ap V2f V4y)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1r \in \\ ((2^{A_27a})^{A_27a}).(\forall V2f \in (A_27a^{A_27a}).(\forall V3x \in A_27a. \\ ((p (ap (ap (ap (c_2Eposet_2Egfp\ A_27a) (ap (ap (c_2Epair_2E_2C \\ (2^{A_27a}) ((2^{A_27a})^{A_27a})) V0s) V1r)) V2f) V3x)) \Leftrightarrow ((p (ap V0s V3x)) \wedge \\ (((ap V2f V3x) = V3x) \wedge (\forall V4y \in A_27a.(((p (ap V0s V4y)) \wedge (p (\\ ap (ap V1r V4y) (ap V2f V4y)))) \Rightarrow (p (ap (ap V1r V4y) V3x)))))))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{60}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& (2^{A.27a}) ((2^{A.27a})^{A.27a})). (\forall V1f \in (A.27a^{A.27a}). (((\\
& p \ (ap \ (c_2Eposet_2Eposet \ A.27a) \ V0p)) \wedge ((p \ (ap \ (c_2Eposet_2Ecomplete \\
& A.27a) \ V0p)) \wedge ((p \ (ap \ (ap \ (ap \ (c_2Eposet_2Efunction \ A.27a \ A.27a) \\
& (ap \ (c_2Eposet_2Ecarrier \ A.27a) \ V0p)) \ (ap \ (c_2Eposet_2Ecarrier \\
& A.27a) \ V0p)) \ V1f)) \wedge (p \ (ap \ (ap \ (c_2Eposet_2Emonotonic \ A.27a) \ V0p) \\
& V1f)))))) \Rightarrow (\exists V2x \in A.27a. (p \ (ap \ (ap \ (ap \ (c_2Eposet_2Egfp \ A.27a) \\
& V0p) \ V1f) \ V2x))))))
\end{aligned}$$