thm_2Eposet_2Elfp__unique (TMXXWjukazSV3QEaJzriSokRxUycC8fbvCe)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $c\_2Eposet\_2Eposet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Eposet\ A\_27a \in (2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))}) \quad (2)$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (3)$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Eposet\_2Elfp : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Elfp\ A\_27a \in (((2^{A\_27a})^{(A\_27a^{A\_27a})})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_2} \tag{4}$$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{10}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a.(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))\ V0s)\ V1r))) \Leftrightarrow ((\exists V2x \in A\_27a.(p\ (ap\ V0s\ V2x))) \wedge ((\forall V3x \in A\_27a.((p\ (ap\ V0s\ V3x)) \Rightarrow (p\ (ap\ (ap\ V1r\ V3x)\ V3x)))) \wedge ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.(((p\ (ap\ V0s\ V4x)) \wedge ((p\ (ap\ V0s\ V5y)) \wedge ((p\ (ap\ (ap\ V1r\ V4x)\ V5y)) \wedge (p\ (ap\ (ap\ V1r\ V5y)\ V4x))))) \Rightarrow (V4x = V5y)))) \wedge (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.(\forall V8z \in A\_27a.(((p\ (ap\ V0s\ V6x)) \wedge ((p\ (ap\ V0s\ V7y)) \wedge ((p\ (ap\ V0s\ V8z)) \wedge ((p\ (ap\ (ap\ V1r\ V6x)\ V7y)) \wedge (p\ (ap\ (ap\ V1r\ V7y)\ V8z)))))) \Rightarrow (p\ (ap\ (ap\ V1r\ V6x)\ V8z))))))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in ((2^{A\_27a})^{A\_27a}).(\forall V2f \in (A\_27a^{A\_27a}).(\forall V3x \in A\_27a.((p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Elfp\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))\ V0s)\ V1r))\ V2f)\ V3x)) \Leftrightarrow ((p\ (ap\ V0s\ V3x)) \wedge (((ap\ V2f\ V3x) = V3x) \wedge (\forall V4y \in A\_27a.(((p\ (ap\ V0s\ V4y)) \wedge (p\ (ap\ (ap\ V1r\ (ap\ V2f\ V4y))\ V4y))) \Rightarrow (p\ (ap\ (ap\ V1r\ V3x)\ V4y))))))))))) \tag{14}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a})).(\forall V1f \in (A\_27a^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Elfp\ A\_27a)\ V0p)\ V1f)\ V2x)) \wedge (p\ (ap\ (ap\ (ap\ (c\_2Eposet\_2Elfp\ A\_27a)\ V0p)\ V1f)\ V3x\_27)))) \Rightarrow (V2x = V3x\_27))))))$$