

thm_2Eposet_2Eposet__antisym (TMdF6xYXHSacYAXED8vRpgS6LetX8xs6x2d)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Eposet_2Eposet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eposet_2Eposet\ A_27a \in (2^{(ty_2Epair_2Eprod\ (2^{A_27a})\ ((2^{A_27a})^{A_27a}))}) \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1r \in \\ & ((2^{A_27a})^{A_27a}).((p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C \\ & (2^{A_27a})\ ((2^{A_27a})^{A_27a}))\ V0s)\ V1r))) \Leftrightarrow ((\exists V2x \in A_27a. \\ & (p\ (ap\ V0s\ V2x))) \wedge ((\forall V3x \in A_27a.((p\ (ap\ V0s\ V3x)) \Rightarrow (p\ (ap\ (\\ & ap\ V1r\ V3x)\ V3x)))) \wedge ((\forall V4x \in A_27a.(\forall V5y \in A_27a.(\\ & ((p\ (ap\ V0s\ V4x)) \wedge ((p\ (ap\ V0s\ V5y)) \wedge ((p\ (ap\ (ap\ V1r\ V4x)\ V5y)) \wedge (p\ (\\ & ap\ (ap\ V1r\ V5y)\ V4x)))) \Rightarrow (V4x = V5y)))) \wedge (\forall V6x \in A_27a.(\forall V7y \in \\ & A_27a.(\forall V8z \in A_27a.(((p\ (ap\ V0s\ V6x)) \wedge ((p\ (ap\ V0s\ V7y)) \wedge \\ & ((p\ (ap\ V0s\ V8z)) \wedge ((p\ (ap\ (ap\ V1r\ V6x)\ V7y)) \wedge (p\ (ap\ (ap\ V1r\ V7y)\ V8z)))))) \Rightarrow \\ & (p\ (ap\ (ap\ V1r\ V6x)\ V8z)))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1r \in \\ ((2^{A_27a})^{A_27a}). (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((\\ (p\ (ap\ (c_2Eposet_2Eposet\ A_27a)\ (ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a}) \\ ((2^{A_27a})^{A_27a})\ V0s)\ V1r))) \wedge ((p\ (ap\ V0s\ V2x)) \wedge ((p\ (ap\ V0s\ V3y)) \wedge \\ ((p\ (ap\ (ap\ V1r\ V2x)\ V3y)) \wedge (p\ (ap\ (ap\ V1r\ V3y)\ V2x)))))) \Rightarrow (V2x = V3y)))))) \end{aligned}$$