

thm\_2Eposet\_2Eposet\_\_nonempty  
(TMQ1gAMx6YehhMcMj8eUXgQf5RiEtDY8xDw)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (V0P)) (V1P)))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) (V1P))))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod A\_27a A\_27b) (V0x V1y))$

Let  $c\_2Eposet\_2Eposet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eposet\_2Eposet\ A\_27a \in (2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))}) \quad (3)$$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in \\ & ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))\ V0s)\ V1r))) \Leftrightarrow ((\exists V2x \in A\_27a. \\ & (p\ (ap\ V0s\ V2x))) \wedge ((\forall V3x \in A\_27a.((p\ (ap\ V0s\ V3x)) \Rightarrow (p\ (ap\ ( \\ & ap\ V1r\ V3x)\ V3x)))) \wedge ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.( \\ & ((p\ (ap\ V0s\ V4x)) \wedge ((p\ (ap\ V0s\ V5y)) \wedge ((p\ (ap\ (ap\ V1r\ V4x)\ V5y)) \wedge (p\ ( \\ & ap\ (ap\ V1r\ V5y)\ V4x)))))) \Rightarrow (V4x = V5y)))) \wedge (\forall V6x \in A\_27a.(\forall V7y \in \\ & A\_27a.(\forall V8z \in A\_27a.(((p\ (ap\ V0s\ V6x)) \wedge ((p\ (ap\ V0s\ V7y)) \wedge \\ & ((p\ (ap\ V0s\ V8z)) \wedge ((p\ (ap\ (ap\ V1r\ V6x)\ V7y)) \wedge (p\ (ap\ (ap\ V1r\ V7y)\ V8z)))))) \Rightarrow \\ & (p\ (ap\ (ap\ V1r\ V6x)\ V8z)))))))))) \end{aligned} \quad (10)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1r \in \\ & ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (c\_2Eposet\_2Eposet\ A\_27a)\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & (2^{A\_27a})\ ((2^{A\_27a})^{A\_27a}))\ V0s)\ V1r))) \Rightarrow (\exists V2x \in A\_27a. \\ & (p\ (ap\ V0s\ V2x)))))) \end{aligned}$$