

thm_2Epowser_2EPOWSER_INSIDEA (TMc5g85FTmEdErCgzPyt6DzRGiGwvosMpGU)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP_CLASS a)))$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (8)$$

Definition 9 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 10 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2)$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (9)$$

Definition 14 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (10)$$

Definition 15 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 16 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{11}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{12}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{13}$$

Definition 17 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{14}$$

Let $c_2Erealax_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod \ ty_2Ehreal)) \tag{15}$$

Definition 18 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 20 We define c_2Ebool_2E2F5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c_2Ebool_2E21 \ 2) \ (\lambda V2t \in 2.$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 22 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap \ (ap \ (ap \ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2ESND \\ A_27a \ A_27b \in (A_27b^{(ty_2Epair_2Eprod \ A_27a \ A_27b)}) \end{aligned} \tag{16}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EFST \\ A_27a \ A_27b \in (A_27a^{(ty_2Epair_2Eprod \ A_27a \ A_27b)}) \end{aligned} \tag{17}$$

Definition 23 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Emetric_2Emetric \ A0) \tag{18}$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Emetric_2Emetric \ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod \ A_27a \ A_27a)}})) \end{aligned} \tag{19}$$

Definition 24 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (20)$$

Definition 25 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Ebounded A_27a A_27b \in ((2^{(A_27a)^{A_27b}})^{(ty_2Epair_2Eprod (ty_2Emetric_2Emetric A_27a) ((2^{A_27b})^{A_27b}))}) \quad (21)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (22)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (23)$$

Definition 26 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Definition 27 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 28 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 29 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 30 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal)^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (24)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (25)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (26)$$

Definition 31 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric A_27a). (ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a)^{A_27b}}) \quad (27)$$

Definition 32 We define $c_Eseq_E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 33 We define $c_Eseq_Econvergent$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_Eseq_E_2D_2D_3E$

Definition 34 We define $c_Eseq_Ecauchy$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_Eseq_E_2D_2D_3E$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (28)$$

Definition 35 We define c_Eseq_Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2Enum_2Enum$

Definition 36 We define $c_Eseq_Esummable$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_Eseq_E_2D_2D_3E$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (29)$$

Definition 37 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (30)$$

Definition 38 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 39 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Assume the following.

$$True \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27b.((ap\ (\lambda V2x \in A_27b.V0t1)\ V1t2) = V0t1))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V0x))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((\neg(p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V0x)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Erealax_2Ereal_lt V0x) V2z))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (\neg(V0x = V1y))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V2z)) \Rightarrow ((p (ap (ap \\
& c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) \\
& (ap (ap c_2Erealax_2Ereal_mul V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap (ap c_2Erealax_2Ereal_mul V1y) V0x)) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) (ap (ap c_2Ereal_2E_2F V2z) V0x))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y))) \Rightarrow \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Ereal_2E_2F V0x) V1y)) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Ereal_2Eabs (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Ereal_2Eabs \\
& V0x))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y)) = \\
& (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Eabs\ V0x))\ (ap\ c_2Ereal_2Eabs \\
& \quad V1y))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ c_2Ereal_2Eabs\ V0x))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Rightarrow ((ap\ c_2Ereal_2Eabs\ (ap\ c_2Erealax_2Einv\ V0x)) = \\
& \quad (ap\ c_2Erealax_2Einv\ (ap\ c_2Ereal_2Eabs\ V0x))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ c_2Ereal_2Eabs\ (ap\ c_2Ereal_2Eabs \\
& \quad V0x)) = (ap\ c_2Ereal_2Eabs\ V0x)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((V0x = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) \vee (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ c_2Ereal_2Eabs\ V0x))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& ((\neg(V0c = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \Rightarrow (\neg((ap \\
& (ap\ c_2Ereal_2Epow\ V0c)\ V1n) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. ((\neg(V0c = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Rightarrow (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Erealax_2Einv \\
& (ap\ (ap\ c_2Ereal_2Epow\ V0c)\ V1n)) = (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Erealax_2Einv \\
& \quad V0c))\ V1n))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Eabs\ V0c))\ V1n) = (ap\ c_2Ereal_2Eabs \\
& \quad (ap\ (ap\ c_2Ereal_2Epow\ V0c)\ V1n))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Epow (ap \\
& (ap c_2Erealax_2Ereal_mul V1x) V2y)) V0n) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2Epow V1x) V0n)) (ap (ap c_2Ereal_2Epow V2y) V0n))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((p (ap (ap \\
& (c_2Enets_2Ebunded ty_2Erealax_2Ereal ty_2Enum_2Enum) (ap \\
& (ap (c_2Epair_2E_2C (ty_2Emetric_2Emetric ty_2Erealax_2Ereal) \\
& ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) c_2Emetric_2Emr1) c_2Earithmic_2E_3E_3D)) \\
& V0s)) \Leftrightarrow (\exists V1k \in ty_2Erealax_2Ereal. (\forall V2n \in ty_2Enum_2Enum. \\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap V0s V2n)) \\
& V1k))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((p (ap c_2Eseq_2Ecauchy \\
& V0f)) \Rightarrow (p (ap (ap (c_2Enets_2Ebunded ty_2Erealax_2Ereal ty_2Enum_2Enum) \\
& (ap (ap (c_2Epair_2E_2C (ty_2Emetric_2Emetric ty_2Erealax_2Ereal) \\
& ((2^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})) c_2Emetric_2Emr1) c_2Earithmic_2E_3E_3D)) \\
& V0f))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((p (ap c_2Eseq_2Ecauchy \\
& V0f)) \Leftrightarrow (p (ap c_2Eseq_2Econvergent V0f))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (\forall V1x0 \in \\
& ty_2Erealax_2Ereal. (\forall V2c \in ty_2Erealax_2Ereal. ((p (ap \\
& (ap c_2Eseq_2Esums V0x) V1x0)) \Rightarrow (p (ap (ap c_2Eseq_2Esums (\lambda V3n \in \\
& ty_2Enum_2Enum. (ap (ap c_2Erealax_2Ereal_mul V2c) (ap V0x V3n)))) \\
& (ap (ap c_2Erealax_2Ereal_mul V2c) V1x0))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). ((p (ap c_2Eseq_2Esummable \\
& V0f)) \Rightarrow (p (ap (ap c_2Eseq_2E_2D_2D_3E V0f) (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).((\exists V2N \in ty_2Enum_2Enum. \\
& (\forall V3n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E_3D \\
& V3n) V2N)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs \\
& (ap V0f V3n))) (ap V1g V3n)))))) \wedge (p (ap c_2Eseq_2Esummable V1g))) \Rightarrow \\
& (p (ap c_2Eseq_2Esummable V0f))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Eabs V0x)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \Rightarrow (p (ap \\
& (ap c_2Eseq_2Esums (\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Ereal_2Epow \\
& V0x) V1n))) (ap c_2Erealax_2Einv (ap (ap c_2Ereal_2Ereal_sub \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL (\\
& ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) V0x))))))
\end{aligned} \tag{66}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1x \in \\
& ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.(((p (\\
& ap c_2Eseq_2Esummable (\lambda V3n \in ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_mul \\
& (ap V0f V3n)) (ap (ap c_2Ereal_2Epow V1x) V3n)))))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Eabs V2z)) (ap c_2Ereal_2Eabs V1x)))))) \Rightarrow (p (ap c_2Eseq_2Esummable \\
& (\lambda V4n \in ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_mul (ap \\
& c_2Ereal_2Eabs (ap V0f V4n))) (ap (ap c_2Ereal_2Epow V2z) V4n)))))))))
\end{aligned}$$