# thm_2Epred__set_2EABS_DIFF_SUM_IMAGE (TMZF8TkWcVmuan3ZSdEqHRYpqf2q9NpBqXh)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

**Definition 7** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

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**Definition 8** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$ (

**Definition 9** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** (*the* $(\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 10** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

**Definition 11** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 12** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 13** *We define* c_2Earithmetic_2EABS__DIFF *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1m \in ty\_2Enu$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (6)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \qquad (7)$$

**Definition 14** *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 15** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 16** *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (8)$$

**Definition 17** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

**Definition 18** *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\text{c\_2Ebool\_2}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

**Definition 19** *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

**Definition 20** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(11)$$

**Definition 21** *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 22** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 23** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

**Definition 24** *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 25** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 26** *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF).$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1)$$

$$(12)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

$$(13)$$

**Definition 27** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(14)$$

**Definition 28** *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 29** *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

**Definition 30** *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (ap$

**Definition 31** *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ (2$

Let $c\_2Epred\_\_set\_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EITSET \\ A\_27a\ A\_27b \in (((A\_27b^{A\_27b})^{(2^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})})$$

$$(15)$$

**Definition 32** *We define* c_2Epred__set_2ESUM__IMAGE *to be* $\lambda A\_27a : \iota.\lambda V0f \in (ty\_2Enum\_2Enum^{A\_27a}$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n))) \tag{16}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO}))) = V0m) \wedge ($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n)))))))))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ ($$
$$ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{18}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{19}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \tag{20}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{21}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Earithmetic\_2EABS\_\_DIFF}$$
$$V0n)\ \mathsf{c\_2Enum\_2E0}) = V0n) \wedge ((ap\ (ap\ \mathsf{c\_2Earithmetic\_2EABS\_\_DIFF}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))) \tag{22}$$

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Assume the following.

$$(\forall V0n1 \in ty\_2Enum\_2Enum.(\forall V1n2 \in ty\_2Enum\_2Enum.$$
$$(\forall V2m1 \in ty\_2Enum\_2Enum.(\forall V3m2 \in ty\_2Enum\_2Enum.$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2EABS\_\_DIFF}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n1)\ V1n2))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V2m1)\ V3m2)))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2EABS\_\_DIFF}$$
$$V0n1)\ V2m1))\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2EABS\_\_DIFF}\ V1n2)\ V3m2))))))))) \tag{23}$$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t)))\wedge(((\neg True){\Leftrightarrow}False)\wedge$$
$$((\neg False){\Leftrightarrow}True))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow}$$
$$(p\ V0t))\wedge(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(\neg(p\ V0t)))\wedge(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}(\neg( \tag{27}$$
$$p\ V0t))))))))$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t){\Rightarrow}False){\Leftrightarrow}((p\ V0t){\Leftrightarrow}False))) \tag{28}$$

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Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
c_2Enum_2E0) $V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)$ c_2Enum_2E0$) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap$ c_2Earithmetic_2ENUMERAL $V2n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V3m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Enumeral_2EiZ $(ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A$ c_2Enum_2E0$)\ V4n) =$ c_2Enum_2E0$))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)$ c_2Enum_2E0$) =$ c_2Enum_2E0$))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap$ c_2Earithmetic_2ENUMERAL $V6n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V7m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
c_2Enum_2E0$)\ V8n) =$ c_2Enum_2E0$))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)$ c_2Enum_2E0$) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap$ c_2Earithmetic_2ENUMERAL $V10n))\ (ap$ c_2Earithmetic_2ENUMERAL
$V11m)) = (ap$ c_2Earithmetic_2ENUMERAL $(ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
$V12n))) =$ c_2Enum_2E0$))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP$ c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Earithmetic_2EBIT2 $V13n))) =$ c_2Enum_2E0$))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)$ c_2Enum_2E0$) =$
$(ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO$))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap$ c_2Earithmetic_2ENUMERAL $V15n))$
$(ap$ c_2Earithmetic_2ENUMERAL $V16m)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap$ c_2Enum_2ESUC
c_2Enum_2E0$) = (ap$ c_2Earithmetic_2ENUMERAL $(ap$ c_2Earithmetic_2EBIT1
c_2Earithmetic_2EZERO$)))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap$ c_2Enum_2ESUC $(ap$ c_2Earithmetic_2ENUMERAL $V17n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Enum_2ESUC $V17n))))\wedge(((ap$ c_2Eprim__rec_2EPRE c_2Enum_2E0$) =$
c_2Enum_2E0$)\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap$ c_2Eprim__rec_2EPRE
$(ap$ c_2Earithmetic_2ENUMERAL $V18n)) = (ap$ c_2Earithmetic_2ENUMERAL
$(ap$ c_2Eprim__rec_2EPRE $V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap$ c_2Earithmetic_2ENUMERAL $V19n) =$ c_2Enum_2E0$)\Leftrightarrow(V19n =$ c_2Earithmetic_2EZERO$)))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.(($c_2Enum_2E0 $= (ap$ c_2Earithmetic_2ENUMERAL
$V20n))\Leftrightarrow(V20n =$ c_2Earithmetic_2EZERO$)))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap$ c_2Earithmetic_2ENUMERAL
$V21n) = (ap$ c_2Earithmetic_2ENUMERAL $V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V23n)$ c_2Enum_2E0$))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Enum_2E0$)\ (ap$ c_2Earithmetic_2ENUMERAL
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Earithmetic_2EZERO$)$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C\ (ap$ c_2Earithmetic_2ENUMERAL
$V25n))\ (ap$ c_2Earithmetic_2ENUMERAL $V26m)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E$
c_2Enum_2E0$)\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap$ c_2Earithmetic_2ENUMERAL
$V28n))$ c_2Enum_2E0$))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$ c_2Earithmetic_2EZERO$)$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\ (ap$ c_2Earithmetic_2ENUMERAL
$V29n))\ (ap$ c_2Earithmetic_2ENUMERAL $V30m)))\Leftrightarrow(p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D$
c_2Enum_2E0$)\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap$ c_2Earithmetic_2E_3C_3D $(ap$ c_2Earithmetic_2ENUMERAL

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ \textsf{c\_2Earithmetic\_2EZERO})\ V0n)) \Leftrightarrow$$
$$True) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ \textsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \textsf{c\_2Earithmetic\_2EZERO})) \Leftrightarrow False) \wedge$$
$$(((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}$$
$$V0n))\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0n)\ V1m)))))))))))$$

(30)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in$$
$$(2^{A\_27a}).((\neg(p\ (ap\ (ap\ (\textsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap$$
$$(ap\ (\textsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V1s)\ V0x) = V1s))))$$

(31)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).(($$
$$(p\ (ap\ V0P\ (\textsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}).$$
$$(((p\ (ap\ (\textsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow$$
$$(\forall V2e \in A\_27a.((\neg(p\ (ap\ (ap\ (\textsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2e)\ V1s))) \Rightarrow$$
$$(p\ (ap\ V0P\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2e)\ V1s)))))))) \Rightarrow$$
$$(\forall V3s \in (2^{A\_27a}).((p\ (ap\ (\textsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)$$
$$V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))$$

(32)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}).$$
$$(((ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}\ A\_27a)\ V0f)\ (\textsf{c\_2Epred\_\_set\_2EEMPTY}$$
$$A\_27a)) = \textsf{c\_2Enum\_2E0}) \wedge (\forall V1e \in A\_27a.(\forall V2s \in (2^{A\_27a}).$$
$$((p\ (ap\ (\textsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}$$
$$A\_27a)\ V0f)\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1e)\ V2s)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ V0f\ V1e))\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}$$
$$A\_27a)\ V0f)\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2EDELETE}\ A\_27a)\ V2s)\ V1e)))))))))$$

(33)

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Enum\_2Enum^{A\_27a}).$$
$$(\forall V1g \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2s \in (2^{A\_27a}).$$
$$((p\ (ap\ (\textsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V2s)) \Rightarrow (p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ \textsf{c\_2Earithmetic\_2EABS\_\_DIFF}\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}$$
$$A\_27a)\ V0f)\ V2s))\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}\ A\_27a)\ V1g)$$
$$V2s)))\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2ESUM\_\_IMAGE}\ A\_27a)\ (\lambda V3x \in A\_27a.$$
$$(ap\ (ap\ \textsf{c\_2Earithmetic\_2EABS\_\_DIFF}\ (ap\ V0f\ V3x))\ (ap\ V1g\ V3x))))$$
$$V2s)))))))$$