

# thm\_2Epred\_set\_2EBIGINTER\_INSERT (TMFavHggio3wXBsSBLi8UgnQbCNn6FaY6Yx)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_7E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_5C$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 8** We define  $c\_2Ebool\_2E\_IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \tag{3}$$

**Definition 10** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EINTER) A\_27a V0s V1t)$

**Definition 11** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21) 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E5C\_2F) V0t1 V1t2 V2t))))$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EINSERT) A\_27a V0x V1s)$

**Definition 13** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EBIGINTER) A\_27a V0P)$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x))))))) \tag{9}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \tag{10}$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (12)$$

Assume the following.

$$2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow 2. (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (13)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}). (\forall V1v \in A_{.27a}. ((\forall V2x \in A_{.27a}. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (14)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1t \in (2^{A_{.27a}}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}. ((p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V2x) V1t))))))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1t \in (2^{A_{.27a}}). (\forall V2x \in A_{.27a}. ((p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V2x) (ap (ap (c_{.2Epred\_set\_2EINTER } A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V2x) V0s)) \wedge (p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V2x) V1t))))))) \quad (16)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1y \in A_{.27a}. (\forall V2s \in (2^{A_{.27a}}). ((p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V0x) (ap (ap (c_{.2Epred\_set\_2EINSERT } A_{.27a}) V1y) V2s))) \Leftrightarrow ((V0x = V1y) \vee (p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V0x) V2s))))))) \quad (17)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. (\forall V1B \in (2^{(2^{A_{.27a}})}). ((p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V0x) (ap (c_{.2Epred\_set\_2EBIGINTER } A_{.27a}) V1B))) \Leftrightarrow (\forall V2P \in (2^{A_{.27a}}). ((p (ap (ap (c_{.2Ebool\_2EIN } (2^{A_{.27a}})) V2P) V1B)) \Rightarrow (p (ap (ap (c_{.2Ebool\_2EIN } A_{.27a}) V0x) V2P))))))) \quad (18)$$

**Theorem 1**

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}). (\forall V1B \in (2^{(2^{A_{.27a}})}). ((ap (c_{.2Epred\_set\_2EBIGINTER } A_{.27a}) (ap (ap (c_{.2Epred\_set\_2EINSERT } (2^{A_{.27a}})) V0P) V1B)) = (ap (ap (c_{.2Epred\_set\_2EINTER } A_{.27a}) V0P) (ap (c_{.2Epred\_set\_2EBIGINTER } A_{.27a}) V1B))))))$$