thm_2Epred__set_2EBIJ__COMPOSE (TMU7Kzz42ebmH6D9whRu2euHy1ALLWDv779)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Epred__set_2ESURJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 11 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 12 *We define* c_2Epred__set_2EBIJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27b}).\ (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27b}).(\forall V4u \in (2^{A\_27c}).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ V3t)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27b\ A\_27c)\ V1g)\ V3t)\ V4u))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V1g)\ V0f))\ V2s)\ V4u))))))) \quad (1)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27b}).\ (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27b}).(\forall V4u \in (2^{A\_27c}).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESURJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ V3t)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESURJ}\ A\_27b\ A\_27c)\ V1g)\ V3t)\ V4u))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESURJ}\ A\_27a\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V1g)\ V0f))\ V2s)\ V4u)))))))) \tag{2}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27b}).\ (\forall V2s \in (2^{A\_27a}).(\forall V3t \in (2^{A\_27b}).(\forall V4u \in (2^{A\_27c}).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ V3t)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27b\ A\_27c)\ V1g)\ V3t)\ V4u))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27c\ A\_27b)\ V1g)\ V0f))\ V2s)\ V4u))))))))$$