thm_2Epred__set_2EBIJ__EMPTY (TM-Vay2KEXKEkednyXYqp5mMpgKsV2xDKAMF)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* c_2Epred__set_2ESURJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 13 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Definition 14 *We define* c_2Epred__set_2EBIJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).((\forall V1s \in (2^{A\_27b}).(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27b)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1s)))\wedge(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b)))\Leftrightarrow(V2s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).((\forall V1s \in (2^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESURJ}\ A\_27a\ A\_27b)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1s))\Leftrightarrow(V1s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b))))\wedge(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESURJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b)))\Leftrightarrow(V2s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in (A\_27b^{A\_27a}).((\forall V1s \in (2^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V1s))\Leftrightarrow(V1s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b))))\wedge(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ V2s)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b)))\Leftrightarrow(V2s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))))$$