

thm_2Epred__set_2ECARD__CROSS
(TMQbm74Vu8xvikdJD1ZCGDZoNKBuJW3DDKG)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^{2^m}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (10)$$

Definition 17 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (11)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (12)$$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 19 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 20 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 21 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 22 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A-27a}).\lambda V1Q \in (2^{A-27b}).(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2ECARD A_27a \in (ty_2Enum_2Enum^{(2^{A-27a})}) \quad (13)$$

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 24 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (15)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \Rightarrow (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))))) \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (37)$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) \\ V0P) V2x) V4y) = (ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) V1Q) V3x_{.27}) \\ V5y_{.27}))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN \\ A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V1t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\neg(p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V0x) (c_{.2E}pred_{.set}_{.2E}EMPTY A_{.27a})))))) \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) \\ V2x) (ap (ap (c_{.2E}pred_{.set}_{.2E}INTER A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap \\ (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V0s)) \wedge (p (ap (ap (c_{.2E}bool_{.2E}IN \\ A_{.27a}) V2x) V1t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in \\ A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V0x) (ap (ap (c_{.2E}pred_{.set}_{.2E}INSERT \\ A_{.27a}) V1y) (c_{.2E}pred_{.set}_{.2E}EMPTY A_{.27a})))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{(2^{A_{.27a}})}).((\\ (p (ap V0P (c_{.2E}pred_{.set}_{.2E}EMPTY A_{.27a})) \wedge (\forall V1s \in (2^{A_{.27a}}). \\ (((p (ap (c_{.2E}pred_{.set}_{.2E}FINITE A_{.27a}) V1s)) \wedge (p (ap V0P V1s))) \Rightarrow \\ (\forall V2e \in A_{.27a}.((\neg(p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2e) V1s))) \Rightarrow \\ (p (ap V0P (ap (ap (c_{.2E}pred_{.set}_{.2E}INSERT A_{.27a}) V2e) V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_{.27a}}).((p (ap (c_{.2E}pred_{.set}_{.2E}FINITE A_{.27a}) \\ V3s)) \Rightarrow (p (ap V0P V3s)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27a})\ (ap\ (ap\ (c_{.2}Epred_set_2EINSERT\ A_{.27a})\ V0x)\ (c_{.2}Epred_set_2EEMPTY\ A_{.27a})))))) \quad (45)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ (c_{.2}Epred_set_2EEMPTY\ A_{.27a})) = c_{.2}Enum_2E0) \quad (46)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (\forall V1x \in A_{.27a}.((ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_{.2}Epred_set_2EINSERT\ A_{.27a})\ V1x)\ V0s)) = (ap\ (ap\ (ap\ (c_{.2}Ebool_2ECOND\ ty_2Enum_2Enum)\ (ap\ (ap\ (c_{.2}Ebool_2EIN\ A_{.27a})\ V1x)\ V0s))\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ V0s))\ (ap\ c_{.2}Enum_2ESUC\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ V0s)))))))))) \quad (47)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (\forall V1t \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27a})\ V1t)) \Rightarrow ((ap\ (ap\ c_{.2}Earithmetic_2E_2B\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_{.2}Epred_set_2EUNION\ A_{.27a})\ V0s)\ V1t)))\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_{.2}Epred_set_2EINTER\ A_{.27a})\ V0s)\ V1t))) = (ap\ (ap\ c_{.2}Earithmetic_2E_2B\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ V0s))\ (ap\ (c_{.2}Epred_set_2ECARD\ A_{.27a})\ V1t)))))))))) \quad (48)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in (2^{A_{.27b}}).(\forall V2x \in (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}).((p\ (ap\ (ap\ (c_{.2}Ebool_2EIN\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})\ V2x)\ (ap\ (ap\ (c_{.2}Epred_set_2ECROSS\ A_{.27a}\ A_{.27b})\ V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_{.2}Ebool_2EIN\ A_{.27a})\ (ap\ (c_{.2}Epair_2EFS\ A_{.27a}\ A_{.27b})\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_{.2}Ebool_2EIN\ A_{.27b})\ (ap\ (c_{.2}Epair_2ESND\ A_{.27a}\ A_{.27b})\ V2x))\ V1Q)))))))))) \quad (49)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty\ A_{.27c} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(((ap\ (ap\ (c_{.2}Epred_set_2ECROSS\ A_{.27a}\ A_{.27b})\ V0P)\ (c_{.2}Epred_set_2EEMPTY\ A_{.27b})) = (c_{.2}Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}))) \wedge ((ap\ (ap\ (c_{.2}Epred_set_2ECROSS\ A_{.27c}\ A_{.27a})\ (c_{.2}Epred_set_2EEMPTY\ A_{.27c}))\ V0P) = (c_{.2}Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_{.27c}\ A_{.27a})))))) \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27b}). (\forall V2x \in \\ & A.27a. ((ap\ (ap\ (c.2Epred_set.2ECROSS\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & \quad A.27a)\ V2x)\ V0P))\ V1Q) = (ap\ (ap\ (c.2Epred_set.2EUNION\ (ty.2Epair.2Eprod \\ & \quad A.27a\ A.27b))\ (ap\ (ap\ (c.2Epred_set.2ECROSS\ A.27a\ A.27b)\ (ap\ (\\ & \quad \quad ap\ (c.2Epred_set.2EINSERT\ A.27a)\ V2x)\ (c.2Epred_set.2EEMPTY \\ & \quad A.27a)))\ V1Q))\ (ap\ (ap\ (c.2Epred_set.2ECROSS\ A.27a\ A.27b)\ V0P) \\ & \quad \quad V1Q)))))) \\ & \hspace{15em} (51) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27b}). (((p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad A.27a)\ V0P)) \wedge (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27b)\ V1Q))) \Rightarrow (p \\ & \quad (ap\ (c.2Epred_set.2EFINITE\ (ty.2Epair.2Eprod\ A.27a\ A.27b)) \\ & \quad (ap\ (ap\ (c.2Epred_set.2ECROSS\ A.27a\ A.27b)\ V0P)\ V1Q)))))) \\ & \hspace{15em} (52) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0x \in A.27a. (\forall V1P \in (2^{A.27b}). ((p\ (ap\ (c.2Epred_set.2EFINITE \\ & \quad A.27b)\ V1P)) \Rightarrow ((ap\ (c.2Epred_set.2ECARD\ (ty.2Epair.2Eprod\ A.27a \\ & \quad A.27b))\ (ap\ (ap\ (c.2Epred_set.2ECROSS\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & \quad A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))\ V1P)) = (ap\ (c.2Epred_set.2ECARD \\ & \quad A.27b)\ V1P)))))) \\ & \hspace{15em} (53) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \hspace{15em} (56) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\ & \hspace{15em} (57) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False))) \quad (58)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p)))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{68}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \forall V0P \in (2^{A_{.27a}}).(\forall V1Q \in (2^{A_{.27b}}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_{.27a})\ V0P)) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_{.27b})\ V1Q))) \Rightarrow ((\\ & ap\ (c_2Epred_set_2ECARD\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}))\ (ap \\ & (ap\ (c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27b})\ V0P)\ V1Q)) = (ap\ (ap\ c_2Earithmetic_2E_2A \\ & (ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ V0P))\ (ap\ (c_2Epred_set_2ECARD \\ & A_{.27b})\ V1Q)))))) \end{aligned}$$