

thm_2Epred__set_2ECARD__DIFF
(TMcZtpxRQedTARkox8K9zrAv6fFEi6779GM)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{4}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(5)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

Definition 12 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

$$c_2Enum_2EZERO_REP \in \omega$$
(6)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(7)

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(8)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(9)

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num c_2Enum_2ESUC_REP)$.

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Epred_set_2EEMPTY : \iota \Rightarrow \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2EZERO : \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(10)

Definition 20 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) \text{ of type } \iota \Rightarrow \iota)$.

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Ebool_2EF : \iota \Rightarrow \iota \Rightarrow \iota) \text{ be given. Assume the following.}$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\
& (2^{A_27a}).(\forall V2x \in A_27a.((p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \\
& V2x) \ (ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t))) \Leftrightarrow ((p \ (ap \\
& (ap \ (c_2Ebool_2EIN \ A_27a) \ V2x) \ V0s)) \wedge (p \ (ap \ (ap \ (c_2Ebool_2EIN \\
& A_27a) \ V2x) \ V1t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\
& (2^{A_27a}).((ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V0s) \ V1t) = (\\
& ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V1t) \ V0s))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}).((ap \ (\\
& ap \ (c_2Epred_set_2EINTER \ A_27a) \ (c_2Epred_set_2EEMPTY \ A_27a)) \\
& V0s) = (c_2Epred_set_2EEMPTY \ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\
& ((ap \ (ap \ (c_2Epred_set_2EINTER \ A_27a) \ V1s) \ (c_2Epred_set_2EEMPTY \\
& A_27a)) = (c_2Epred_set_2EEMPTY \ A_27a))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((ap \ (ap \\
& (c_2Epred_set_2EDIFF \ A_27a) \ V0s) \ (c_2Epred_set_2EEMPTY \ A_27a)) = \\
& V0s))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EINTER \\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ V1s))\ V2t) = \\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (2^{A_27a}))\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V0x)\ V2t))\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ (\\ ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ V1s)\ V2t)))\ (ap\ (ap\ (c_2Epred_set_2EINTER \\ A_27a)\ V1s)\ V2t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ (2^{A_27a}). ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap \\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V1s)\ V0x) = V1s)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((ap\ (ap\ (c_2Epred_set_2EDIFF \\ A_27a)\ V0s)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2x)\ V1t)) = \\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EDELETE \\ A_27a)\ V0s)\ V2x))\ V1t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((ap\ (ap\ (c_2Epred_set_2EINTER \\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V0s)\ V2x))\ V1t) = \\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\ A_27a)\ V0s)\ V1t))\ V2x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((\\ (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). \\ ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s))) \Rightarrow \\ (\forall V2e \in A_27a. ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\ (\forall V3s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ \\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EDELETE \\ A_27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (\forall V1t \in (2^{A_{.27a}}). \\ (p\ (ap\ (c_2Epred_set_2EFINITE\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EINTER \\ A_{.27a})\ V0s)\ V1t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((ap\ (c_2Epred_set_2ECARD\ A_{.27a}) \\ (c_2Epred_set_2EEMPTY\ A_{.27a})) = c_2Enum_2E0) \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (\forall V1x \in A_{.27a}.((\\ ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EINSERT \\ A_{.27a})\ V1x)\ V0s)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum) \\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V1x)\ V0s))\ (ap\ (c_2Epred_set_2ECARD \\ A_{.27a})\ V0s))\ (ap\ c_2Enum_2ESUC\ (ap\ (c_2Epred_set_2ECARD\ A_{.27a}) \\ V0s)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_{.27a})\ V0s)) \Rightarrow (\forall V1x \in A_{.27a}.((\\ ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EDELETE \\ A_{.27a})\ V0s)\ V1x)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum) \\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V1x)\ V0s))\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ (ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ V0s))\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ (ap\ (c_2Epred_set_2ECARD \\ A_{.27a})\ V0s)))))) \end{aligned} \quad (37)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0t \in (2^{A_{.27a}}).((p\ (ap \\ (c_2Epred_set_2EFINITE\ A_{.27a})\ V0t)) \Rightarrow (\forall V1s \in (2^{A_{.27a}}). \\ ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_{.27a})\ V1s)) \Rightarrow ((ap\ (c_2Epred_set_2ECARD \\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_{.27a})\ V1s)\ V0t)) = (ap\ (ap \\ c_2Earithmetic_2E_2D\ (ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ V1s)) \\ (ap\ (c_2Epred_set_2ECARD\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EINTER \\ A_{.27a})\ V1s)\ V0t)))))) \end{aligned}$$