

# thm\_2Epred\_\_set\_2ECARD\_INSERT (TMVZX- oiB44UKa1xgDZkMsbdd8BTgkhwmTwe)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V1f \in 2.V1f) V0x)))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap (ap (c\_2Emin\_2E\_3D (2^{2^m})) (\lambda V1f \in 2.V1f) V0x)))$

**Definition 5** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda a.\lambda V1f \in (2^{A-27a}).(ap V1f V0x))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t) V1t2)))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 11** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (5)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b}})^{A\_27a}) \quad (6)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (7)$$

**Definition 13** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2$

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF).$

**Definition 15** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_21 2)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (8)$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP).$

Let  $c\_2Epred\_set\_2ECARD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Epred\_set\_2ECARD A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})}) \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (((ap (c\_2Epred\_set\_2ECARD A\_27a) \\ & (c\_2Epred\_set\_2EMPTY A\_27a)) = c\_2Enum\_2E0) \wedge (\forall V0s \in \\ & (2^{A\_27a}). ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) V0s)) \Rightarrow (\forall V1x \in \\ A\_27a. ((ap (c\_2Epred\_set\_2ECARD A\_27a) (ap (ap (c\_2Epred\_set\_2EINSERT \\ A\_27a) V1x) V0s)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\ (ap (ap (c\_2Ebool\_2EIN A\_27a) V1x) V0s)) (ap (c\_2Epred\_set\_2ECARD \\ A\_27a) V0s)) (ap c\_2Enum\_2ESUC (ap (c\_2Epred\_set\_2ECARD A\_27a) \\ V0s)))))))))) \end{aligned} \quad (10)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). ((p (ap \\ & (c\_2Epred\_set\_2EFINITE A_{27a}) V0s)) \Rightarrow (\forall V1x \in A_{27a}. (( \\ & ap (c\_2Epred\_set\_2ECARD A_{27a}) (ap (ap (c\_2Epred\_set\_2EINSERT \\ & A_{27a}) V1x) V0s)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\ & (ap (ap (c\_2Ebool\_2EIN A_{27a}) V1x) V0s)) (ap (c\_2Epred\_set\_2ECARD \\ & A_{27a}) V0s)) (ap c\_2Enum\_2ESUC (ap (c\_2Epred\_set\_2ECARD A_{27a}) \\ & V0s)))))))) \end{aligned}$$