

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (4)$$

Definition 11 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_2E$

Definition 12 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_2D_2D_2E V0t) c_2Ebool_2E_2E$ Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\neg (V0s = (c_2Epred_set_2EEMPTY A_27a))) \Rightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) (ap (c_2Epred_set_2ECHOICE A_27a) V0s)) V0s))) \quad (7)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (ap (ap (c_2Epred_set_2EINSERT A_27a) V1y) (c_2Epred_set_2EEMPTY A_27a)))) \Leftrightarrow (V0x = V1y)))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\neg ((ap (ap (c_2Epred_set_2EINSERT A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a)) = (c_2Epred_set_2EEMPTY A_27a)))) \quad (9)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Epred_set_2ECHOICE A_27a) (ap (ap (c_2Epred_set_2EINSERT A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a))) = V0x))$$