

thm_2Epred_set_2ECOUNTABLE_NUM
(TMKPf3LSyjXitmSMi35M687cD1u6zJSdghF)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Ecombin_2E_2K$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 9 We define $c_2Ecombin_2E_2S$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 10 We define $c_2Ecombin_2E_2I$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a}) A$

Definition 11 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Definition 15 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Definition 16 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E3F$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{4}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((p (ap \\ & (c_2Epred_set_2Ecountable A_27a) V0s)) \Leftrightarrow (\exists V1f \in (A_27a^{ty_2Enum_2Enum}). \\ & (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Rightarrow \\ & (\exists V3n \in ty_2Enum_2Enum.((ap V1f V3n) = V2x)))))) \end{aligned} \tag{10}$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Enum_2Enum}).(p (ap (c_2Epred_set_2Ecountable ty_2Enum_2Enum) V0s)))$$