

thm_2Epred__set_2ECROSS__SINGS
(TMaDYTT2GigEJANwt3e6jGMhNpgx7k9WoWP)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$) of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P) V0P) V0P))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (V0x V1y))$

Definition 9 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 11 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(c_2Ebool_2E_7E 2) V0t1) V1t2) V2t) V0t1) V1t2) V2t1$.
Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (3)$$

Definition 13 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_7E 2) V0x) V1s$.
Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (4)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (5)$$

Definition 14 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap (ap (c_2Ebool_2E_7E 2) V0P) V1Q)$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap (c_2Ebool_2E_3D_3D_3E V0t) c_2Ebool_2E_7E 2) V0t) V0t)$.
Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in \\
& A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\
& (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\
& (p \ V0A))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\
& p \ V0A)) \vee (\neg(p \ V1B)))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V0x)) = V0x)) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2ESND\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.(\forall V2s \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27b}).(\forall V2x \in \\ & (ty_2Epair_2Eprod\ A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ & A_27a\ A_27b)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b) \\ & V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ (c_2Epair_2EFST \\ & A_27a\ A_27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V2x))\ V1Q))))))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.((p \vee 0A) \Rightarrow ((\neg(p \vee 0A)) \Rightarrow \text{False}))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \vee 0A) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \vee 0A)) \vee (p \vee 1B))) \Rightarrow \text{False}) \Leftrightarrow ((p \vee 0A) \Rightarrow ((\neg(p \vee 1B)) \Rightarrow \text{False})))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \vee 0A)) \Rightarrow \text{False}) \Rightarrow (((p \vee 0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Leftrightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee ((p \vee 1q) \vee (p \vee 2r))) \wedge (((p \vee 0p) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 1q)))) \wedge (((p \vee 1q) \vee ((\neg(p \vee 2r)) \vee (\neg(p \vee 0p)))) \wedge ((p \vee 2r) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \wedge (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee ((\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \vee (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \vee 0p) \Leftrightarrow (p \vee 1q) \Rightarrow (p \vee 2r)) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge (((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge ((\neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p)))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p)))))) \quad (38)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap\ (ap\ (c_2Epred_set_2ECROSS \\ & \quad A_{.27a}\ A_{.27b})\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_{.27a})\ V0x)\ (c_2Epred_set_2EEMPTY \\ & \quad A_{.27a})))\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_{.27b})\ V1y)\ (c_2Epred_set_2EEMPTY \\ & \quad A_{.27b}))) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ (ty_2Epair_2Eprod \\ & \quad A_{.27a}\ A_{.27b}))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_{.27a}\ A_{.27b})\ V0x)\ V1y))\ (c_2Epred_set_2EEMPTY \\ & \quad (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})))))) \end{aligned}$$